Chapter 3

(Lectures 12, 13 and 14)

Longitudinal stick–free static stability and control

Keywords : Hinge moment and its variation with tail angle, elevator deflection and tab deflection ; floating angle of elevator ; stick-free static stability and stick-free neutral point; stick force and stick force gradient and their variations with flight speed ; determination of stick-free neutral point from flight tests.

Topics

Chapter 3

Lecture 12

Longitudinal stick–free static stability and control – 1

Topics

3.1 Introduction

In the analysis of stick-fixed longitudinal static stability it is assumed that the elevator deflection remains constant even after the disturbance. The analysis of the longitudinal static stability when the elevator is free to rotate about its hinge line is called stick-free stability. The flight condition in which this may occur is explained below.

To fly the airplane at different speeds and altitudes, appropriate values of lift coefficient (C_L) are needed, e.g. in level flight, $L = W = (1/2)\ \rho\ V^2\ S\ C_L$ or

$$C_L = 2W / \{(1/2)\rho V^2 S\}$$

As seen in section 2.12.3, different values of δ_{trim} are needed to bring the airplane in equilibrium at each C_L. To hold the elevator at this δ_{trim}, the pilot has to exert a force called stick force (F) at the control stick. $F = GH_e$ where H_e is the hinge moment at the control surface hinge and G is the gearing ratio which depends on the mechanism between stick and the control surface. Figure 3.1 shows a schematic arrangement of the control surfaces and stick.

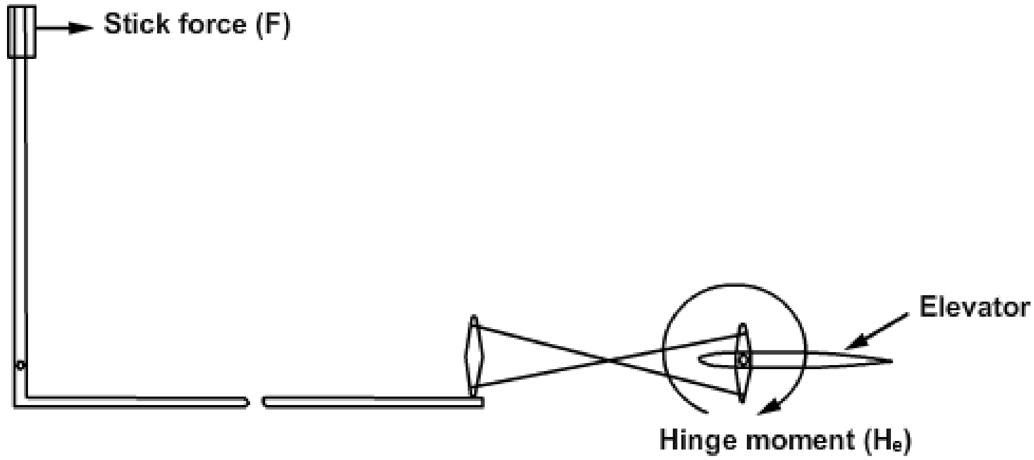

Fig.3.1 Schematic arrangement of elevator and stick Hinge moment and stick force are also shown

To relieve the pilot of the strain of applying the stick force (F) all the time, tab is used to bring the hinge moment to zero. To appreciate the action of a tab note its location as shown in Figs.2.16 a and b. It is observed that by deflecting the tab in a direction opposite to that of the elevator, a lift ΔL_{tab} would be produced. This would slightly reduce the lift due to the elevator, $\Delta L_{\delta e}$, but can make the hinge moment zero. This type of tab is called a trim tab. After applying an appropriate tab deflection, the pilot can leave the stick, i.e. stick is left free as hinge moment is zero. The analysis of stability when the stick is free or the elevator is free to move after the disturbance, is called stick-free stability analysis. It will be explained later that this analysis also facilitates study of aspects like stick force and stick force gradient.

The difference between stability when the stick is fixed and when the stick is free can be explained as follows.

(1) When an airplane flying at an angle of attack α encounters a disturbance, its angle of attack changes to $(\alpha+\Delta\alpha)$. Consequently, the angle of attack of the tail also changes along with that of the airplane. Now, the pressure distribution on the elevator depends on the angles of attack of tail (α_t) , elevator deflection (δ_e) and tab deflection (δ_t). Hence, the moment about the elevator hinge line also depends on these three parameters viz. α_t ,δ_e and δ_t. Thus, when α_t changes as a result of the disturbance, the hinge moment also changes. In stick-free case,

the elevator is not restrained by the pilot, and it automatically takes a position such that the hinge moment is zero (see item 2 below).

(2) It may be pointed out here, that a surface which is free to move about a hinge will always take such a position that the moment about the hinge is zero. For example, consider the rod OB which is hinged at O and attached to another rod OA resting on a table (Fig.3.2). When left to itself, the rod OB will take the vertical position. In this position, the weight of the rod passes through the hinge and the moment about the hinge is zero. If the rod is to be held in another position OB′ (shown by dotted lines in Fig.3.2), then a force 'F' has to be applied to overcome the moment about the hinge which is due to the weight of the rod. However, an important difference between the motion of the rod and that of the elevator must be noted. In the case of the rod the moment about the hinge is due to the weight of the rod. Whereas the moment about the elevator hinge is mainly aerodynamic in nature i.e. due to the pressure distribution on the elevator which depends on α_t, δ_e and δ_t. The influence of the weight of the elevator on stability is discussed in stick free dynamic stability (section 8.15).

(3) Finally, the deflection of an elevator, which is free to move, depends on the disturbance viz. $\Delta\alpha$. This would produce change in the tail lift and consequently, moment about c.g.. Thus, an additional change in C_{mcg} is brought about when the elevator is free. This also results in change in $C_{m\alpha}$ and hence the longitudinal static stability.

The expression for the hinge moment in terms of α_t, δ_e and δ_t, is obtained in section 3.2. The changes in longitudinal static stability are discussed in section 3.3.

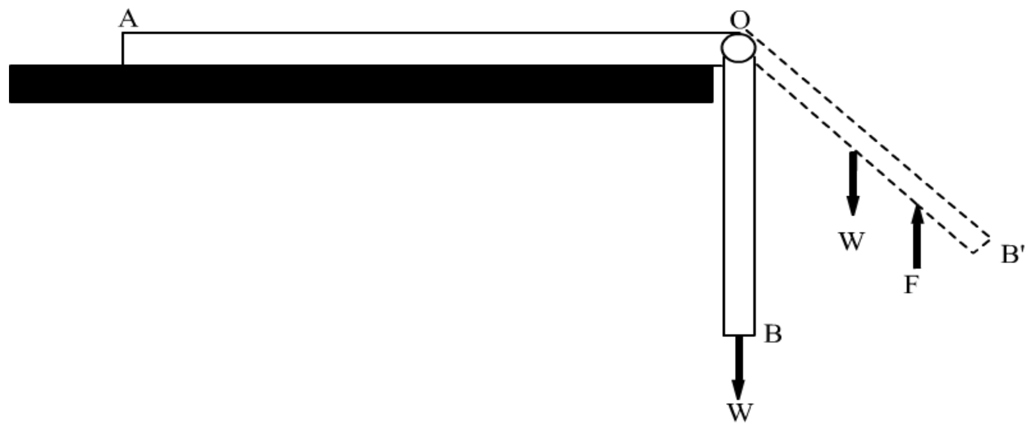

Fig.3.2 Equilibrium positions of a hinged rod

3.2 Hinge moment

To analyze stick-free stability, the dependence of the hinge moment, on α_t, δ_e and δ_t needs to be arrived at. As mentioned earlier, the hinge moment (H_e) is the moment about the control surface hinge due to the pressure distribution on it (control surface). The Hinge moment coefficient (C_{he}) is defined as:

$$H_e = \frac{1}{2}\rho V^2 S_e \bar{c}_e C_{he} \qquad (3.1)$$

$$\text{Or } C_{he} = \frac{H_e}{\frac{1}{2}\rho V^2 S_e \bar{c}_e} \qquad (3.2)$$

where, S_e is the area of elevator aft of the hinge line and $\bar{c}_e$ is the m.a.c. of the elevator area aft of the hinge line. By convention, nose up hinge moment is taken as positive (Fig.3.1).

3.2.1 Changes in the hinge moment due to α_t and δ_e

To examine the effects of α_t and δ_e, consider the changes in pressure distribution on the tail due to these two angles. Figure 3.3a shows the distribution of pressure coefficient (C_p) in potential flow past a symmetric airfoil at zero angle of attack. It may be recalled that C_p is defined as:

$$C_p = \frac{p - p_\infty}{\frac{1}{2}\rho V_\infty^2} \qquad (3.3)$$

where, p = local static pressure, p_∞ = free stream static pressure and

(1/2) ρV_∞^2 = free stream dynamic pressure.

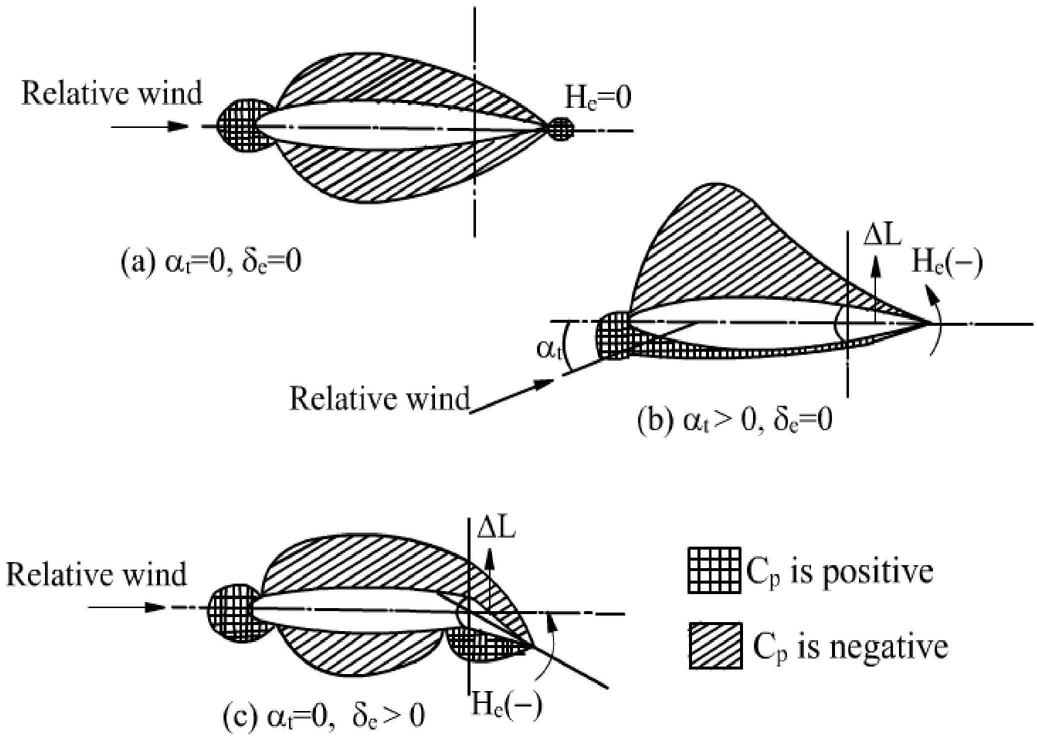

Fig.3.3 Changes in distribution of pressure coefficient with α_t and δ_e (Adapted from “Dommasch, D.O., Sherby, S.S., Connolly, T.F. “Airplane aerodynamics”, Chapter 12 with permission from Pearson Education, Copyright C 1967)

Since, the airfoil used for the tail is symmetric, the C_p distribution, at $\alpha = 0$, is symmetric both on the airfoil and the elevator. Hence, there is no force on the elevator and no hinge moment about the elevator hinge. Figure 3.3b shows the same airfoil at a positive angle of attack (α_t). The distribution of C_p shows that it is negative on the upper surface and positive on the lower surface of the elevator. This results in a positive ΔL_e on the elevator and a nose down (i.e. negative) hinge moment. The hinge moment becomes more negative as α_t increases. Note that $\alpha < \alpha_{stall}$. Figure 3.3c shows the changes in C_p distribution due to positive elevator deflection. This also causes C_p to be negative on the upper surface and positive on the lower surface of the elevator, resulting in positive ΔL_e and negative hinge moment. The effect of the deflection of tab (δ_t) on C_{he} is similar in trend as that due to the elevator.

Typical variations of C_{he} with α_t and δ_e are shown in Fig.3.4. Note that when $\alpha > \alpha_{stall}$ the curves become non-linear.

In the linear region of the curves, C_{he} can be expressed as:

$$C_{he} = C_{h0} + C_{h\alpha t}\,\alpha_t + C_{h\delta e}\,\delta_e + C_{h\delta t}\,\delta_t \tag{3.4}$$

C_{h0} is zero for a symmetric airfoil and is omitted in subsequent discussion;

$C_{h\alpha t} = \partial C_{he} / \partial \alpha_t$; $C_{h\delta e} = \partial C_{he} / \partial \delta_e$; and $C_{h\delta t} = \partial C_{he} / \partial \delta_t$.

The quantities $C_{h\alpha t}$, $C_{h\delta e}$ and $C_{h\delta t}$ depend on the shape of the control surface, area behind the hinge line and the gap between the main surface and the control surface. They are generally negative. Discussion on $C_{h\alpha t}$ and $C_{h\delta e}$ is taken up in chapter 6 after lateral static stability is also covered.The discussion is common for elevator, rudder and aileron.

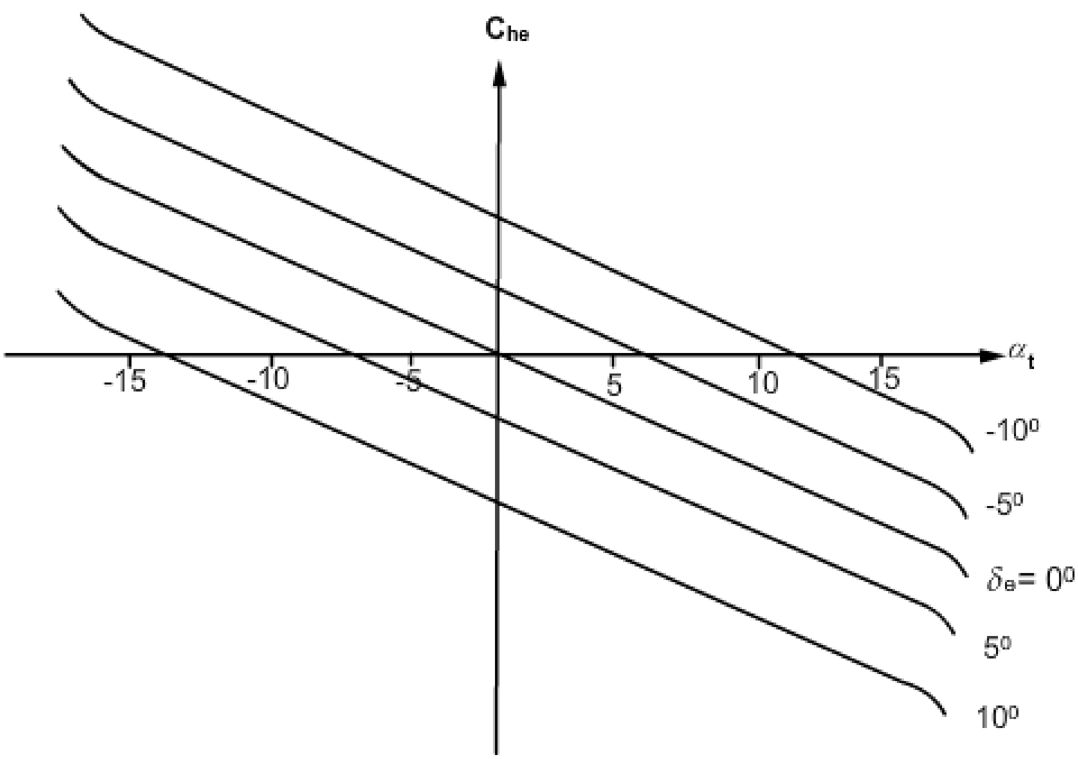

Fig.3.4 Variation of hinge moment coefficient with α_t and δ_e

3.3 Analysis of stick-free static stability

For the analysis of static stability with stick free, it is assumed that as soon as the disturbance is encountered, the elevator takes such a position such that the hinge moment is zero. As pointed out earlier, whenever a surface is free to rotate about a hinge line, it takes such a position such that the hinge moment is zero.

3.3.1 Floating angle of elevator (δ_{efree})

The elevator deflection when the hinge moment is zero is called the floating angle and is denoted by δ_{efree}. It can be obtained by equating *l*.h.s. of Eq.(3.4) to zero. i.e.

$$0 = C_{h\alpha t}\,\alpha_t + C_{h\delta e}\,\delta_{efree} + C_{h\delta t}\,\delta_t \qquad (3.5)$$

$$\delta_{efree} = -\frac{C_{h\alpha t}\,\alpha_t + C_{h\delta t}\,\delta_t}{C_{h\delta e}} \qquad (3.6)$$

3.3.2 Static stability level in stick-free case $(dC_{mcg}/d\alpha)_{stick\text{-}free}$

Assuming the elevator to have attained δ_{efree}, the lift on the tail becomes:

$$C_{Lt} = C_{L\alpha t}\,\alpha_t + \frac{\partial C_L}{\partial \delta_e}\,\delta_{efree} + \frac{\partial C_L}{\partial \delta_t}\,\delta_t$$

$$= C_{L\alpha t}\,\{\alpha_t + \tau\,\delta_{efree} + \tau_{tab}\,\delta_t\} \qquad (3.7)$$

$$\tau_{tab} = \frac{\left(\dfrac{\partial C_L}{\partial \delta_t}\right)}{C_{L\alpha t}} \qquad (3.8)$$

Substituting for δ_{efree}, Eq.(3.7) becomes:

$$C_{Lt} = C_{L\alpha t}\left(\alpha_t - \tau\,\frac{C_{h\alpha t}\,\alpha_t + C_{h\delta t}\,\delta_t}{C_{h\delta e}} + \tau_{tab}\,\delta_t\right)$$

$$= C_{L\alpha t}\,\alpha_t\left(1 - \frac{C_{h\alpha t}}{C_{h\delta e}}\tau\right) - C_{L\alpha t}\left(\frac{C_{h\delta t}}{C_{h\delta e}}\tau - \tau_{tab}\,\delta_t\right) \qquad (3.9)$$

Substituting α_t from Eq.(2.45) in Eq.(3.9) gives:

$$C_{Lt} = C_{L\alpha t}\left(1 - \frac{C_{h\alpha t}}{C_{h\delta e}}\tau\right)\left(\alpha - \varepsilon_0 - \frac{d\varepsilon}{d\alpha}\alpha + i_t\right) - C_{L\alpha t}\left(\tau\frac{C_{h\alpha t}}{C_{h\delta e}}\delta_t - \tau_{tab}\,\delta_t\right) \qquad (3.10)$$

Denoting the moment about c.g. by the tail, in the stick-free case, by C'_{mcgt} it can be expressed as :

$C'_{mcgt} = -V_H\,\eta\,C_{Lt}$

Substituting for C_{Lt} from Eq.(3.10) gives:

$$C'_{mcgt} = V_H\ \eta\ C_{L\alpha t}(\varepsilon_0 - i_t) - V_H\ \eta\ C_{L\alpha t}[\alpha\ (1-\frac{d\varepsilon}{d\alpha}) + \tau_{tab}\ \delta_t]$$

$$-\frac{\tau}{C_{h\delta e}}\ [C_{h\alpha t}\{\ \alpha - \varepsilon_0 - \frac{d\varepsilon}{d\alpha}\alpha + i_t\} + C_{h\delta t}\ \delta_t]$$

Or $C'_{mcgt} = V_H\ \eta\ C_{L\alpha t}\,(\varepsilon_0 - i_t - \tau_{tab}\ \delta_t)(1-\tau\frac{C_{h\alpha t}}{C_{h\delta e}}) - V_H\ \eta\ C_{L\alpha t}\ \alpha(1-\frac{d\varepsilon}{d\alpha})(1-\tau\frac{C_{h\alpha t}}{C_{h\delta e}})$ (3.11)

Differentiating Eq.(3.11) with α and denoting the contribution, of tail to stick-free stability, by $C'_{m\alpha t}$ gives:

$$C'_{m\alpha t} = -V_H\ \eta\ C_{L\alpha t}\ (1-\frac{d\varepsilon}{d\alpha})(1-\tau\frac{C_{h\alpha t}}{C_{h\delta e}})$$

Noting that $C_{m\alpha t} = -V_H\ \eta\ C_{L\alpha t}\,(1-\frac{d\varepsilon}{d\alpha})$, yields :

$$C'_{m\alpha t} = -\ V_H\ \eta\ C_{L\alpha t}\ \ (1-\frac{d\varepsilon}{d\alpha})f = C_{m\alpha t}\ f \tag{3.12}$$

$$f = (1-\tau\frac{C_{h\alpha t}}{C_{h\delta e}}) \tag{3.13}$$

'f' is called free elevator factor.

The contributions of wing, fuselage, nacelle and power do not change by freeing the stick, hence,

$$(C_{m\alpha})_{stick\text{-}free} = C'_{m\alpha} = (C_{m\alpha})_w\ + (C_{m\alpha})_{f,n,p} - V_H\ \eta\ C_{L\alpha t}\ (1-\frac{d\varepsilon}{d\alpha})\ f$$

$$C'_{m\alpha} = C_{L\alpha w}(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}) + (C_{m\alpha})_{f,n,p} - V_H\ \eta\ C_{L\alpha t}\ (1-\frac{d\varepsilon}{d\alpha})\ f \tag{3.14}$$

$$(\frac{dC_m}{dC_L})_{stick\text{-}free} = \frac{1}{C_{L\alpha w}}C'_{m\alpha} = \frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}} + \frac{1}{C_{L\alpha w}}(C_{m\alpha})_{f,n,p} - V_H\,\eta\,\frac{C_{L\alpha t}}{C_{L\alpha w}}(1-\frac{d\varepsilon}{d\alpha})\ f$$

$$= (\frac{dC_m}{dC_L})_{stick\text{-}fixed} + \frac{C_{h\alpha t}}{C_{h\delta e}}\frac{C_{L\alpha t}}{C_{L\alpha w}}\bar{V}_H\,\eta\,\tau(1-\frac{d\varepsilon}{d\alpha}) \tag{3.14a}$$

3.3.3 Neutral point stick-free ($X'_{NP}/\bar{c}$)

In the stick free case, the neutral point is denoted (Ref.1.1, chapter 2) by x'_{NP}. It is obtained by setting $C'_{m\alpha} = 0$

Or $$\frac{x'_{NP}}{\bar{c}} = \frac{x_{ac}}{\bar{c}} - \frac{(C_{m\alpha})_{f,n,p}}{C_{L\alpha w}} - \frac{C_{L\alpha t}}{C_{L\alpha w}} V_H \eta \left(1 - \frac{d\varepsilon}{d\alpha}\right) f \qquad (3.15)$$