Chapter 4

Longitudinal static stability and control – Effect of acceleration

(Lecture 15)

Keywords : Elevator required in pull-up; stick-fixed maneuver point; stick force gradient in pull-up; maneuver point stick-free; overall limits on c.g. travel.

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Example 4.3

4.1 Introduction

An accelerated flight occurs when an airplane (a) has acceleration or deceleration along a straight line (accelerated level flight or climb) or (b) performs maneuvers like loop and turn. In the case of accelerated flight along a straight line, the stability and control equations are the same as those for the unaccelerated flight. However, the engine thrust would be different in accelerated and unaccelerated flights. This difference in engine thrust would result in slightly different contribution of power to C_{mo} and $C_{m\alpha}$. Significant changes in stability and control take place when an airplane goes through a maneuver.

Remark:

In European books, the word maneuver is spelt as "manoeuvre".

4.2 Additional elevator deflection in pull-up

Consider an airplane at the bottom of a loop as shown in Fig.4.1.
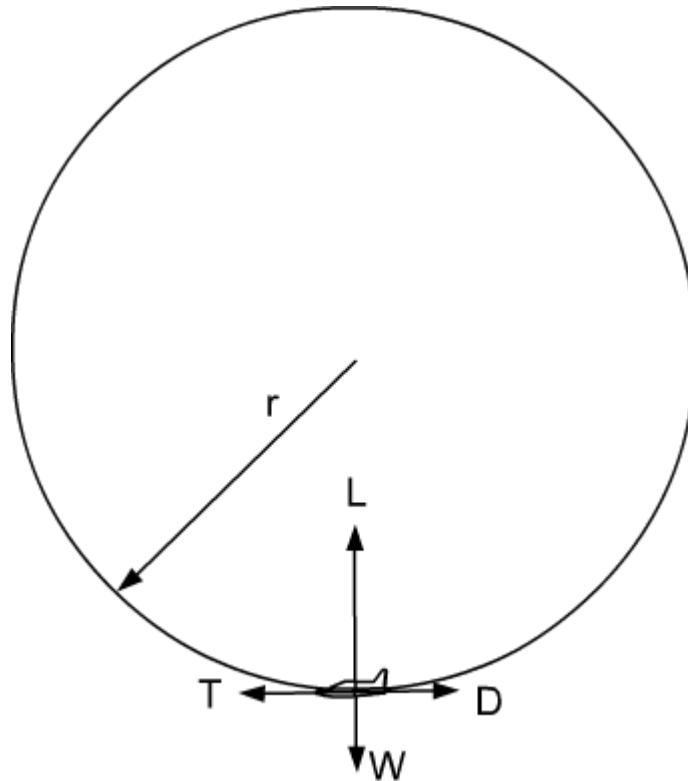
Note : The flight path is circular. Please adjust the resolution of your monitor so that the flight path looks circular.

Fig. 4.1 Airplane in a loop

Let, the flight velocity, load factor and the radius of the loop be V, n and r respectively. Now, L = n W. Further, let ΔL be the excess of lift over that in level flight. Then, ΔL = (n-1) W.

The equations of motion in the plane of symmetry are:

$$T - D = 0 \tag{4.1}$$

$$L - W = (W/g)(V^2/r) = (W/g)\, V\omega \tag{4.2}$$

where, ω is the angular velocity.

Equation (4.2) gives $\omega = (n - 1)\, g / V$ (4.3)

Remarks:

Following three important points need to be noted.

i) As the airplane goes round the loop once, it also goes once around its c.g. To explain this, Fig.4.2 shows the airplane at different points during the loop. It is evident that while completing the loop the airplane has gone around itself once. Thus q, the angular velocity about y-axis, is equal to ω i.e.
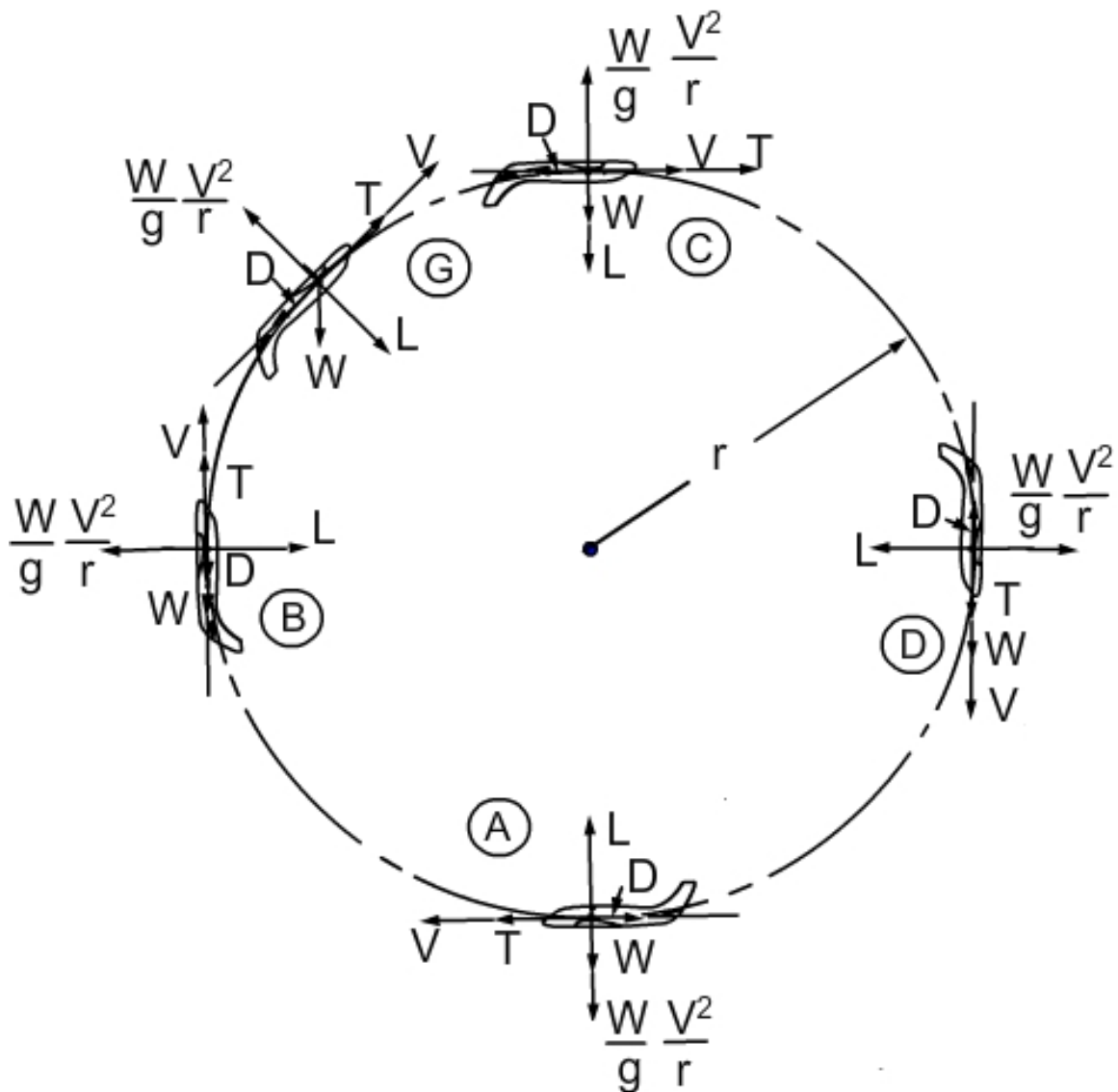
$$q = \omega = (n-1)g / V \tag{4.4}$$

Note : The flight path is circular. Please adjust the resolution of your monitor so that the flight path looks circular.

Fig. 4.2 Airplane attitude at different points in a loop

ii) As the airplane rotates with angular velocity q, the tail which is located at a distance of l_t from c.g., is subjected to a downward velocity $\Delta v_w = q\, l_t$ (Fig.4.3). Thus, the tail is subjected to a relative wind in the upward direction of magnitude $q\, l_t$. This causes a change in the angle of attack of the tail (Fig.4.3) by $\Delta\alpha_t$ given by:

$$\Delta\, \alpha_t = \frac{q\, l_t}{V} = 57.3\frac{q\, l_t}{V} \text{ in degrees} \tag{4.5}$$
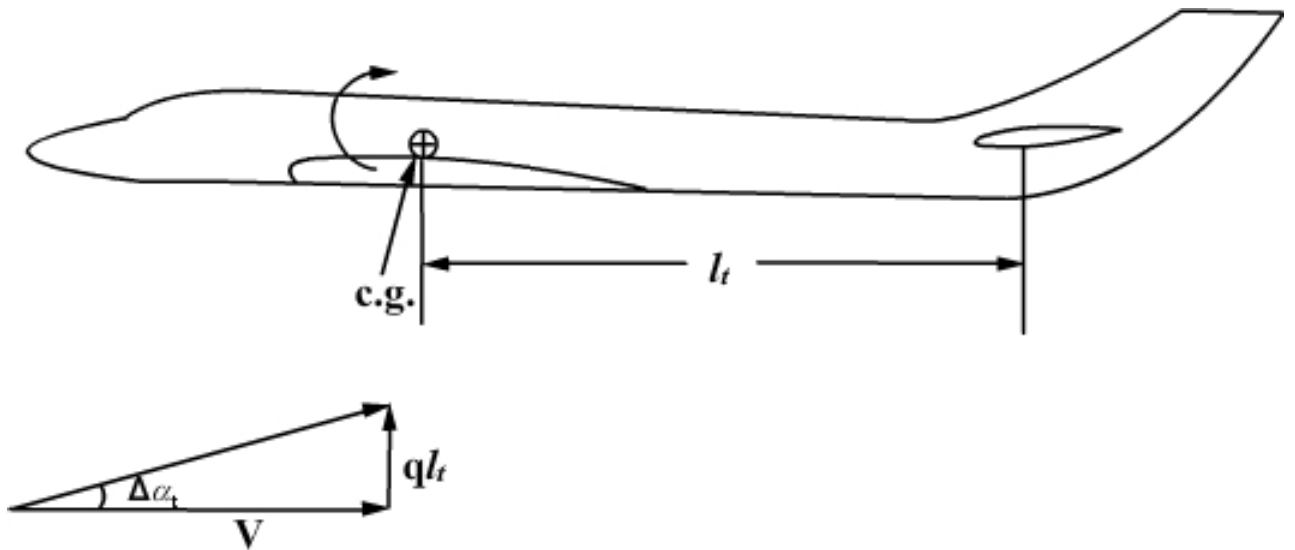

Fig.4.3 changes in α_t in pull-up

iii) This change in $\Delta\alpha_t$ results in lift ΔL_t on tail and negative ΔC_{mcgt} about the c.g. To balance this ΔC_{mcgt}, an additional elevator deflection is needed. Since, the effect of going through a loop is to cause a resisting moment; this effect is called damping in loop. Let, $\Delta\delta_e$ be the additional elevator deflection needed to balance $\Delta\alpha_t$. Then

$$\Delta\delta_e = \frac{-\Delta\alpha_t}{\tau} = -57.3\frac{(n-1)g\, l_t}{V^2\tau} \tag{4.6}$$

The other components of the airplane also experience changes in angle of attack due to the angular velocity in loop. The net effect is approximately accounted for (Ref.1.7, chapter 7) by multiplying Eq.(4.6) by 1.1 i.e.

$$\Delta\delta_e = 1.1 \text{x} (- 57.3)(n-1)g \, l_t / (V^2 \, \tau) = - 63(n-1) \, g \, l_t / (V^2 \tau) \tag{4.7}$$

The pull-up flight can be considered as a part of a loop. Hence, combining Eqs.(2.84)and (4.7) , the elevator deflection in pull-up $(\delta_e)_{pullup}$ is given by:

$$(\delta_e)_{pull\text{-}up} = \delta_{eoCL} - C_L \frac{(dC_m / dC_L)_{stick\text{-}fixed}}{C_{m\delta}} - \frac{63 \, (n-1) \, g \, l_t}{\tau \, V^2} \tag{4.8}$$

In a pull-up with load factor of n, L = n W. Hence,

$$C_L = 2nW / \rho V^2 S$$

Hence, $(\delta_e)_{pull\text{-}up} = \delta_{eoCL} - \frac{2nW}{\rho V^2 S} \frac{(dC_m / dC_L)_{stick\text{-}fixed}}{C_{m\delta}} - \frac{63 \, (n-1) g \, l_t}{\tau \, V^2}$ \hfill (4.9)

4.3 Elevator angle per g:

The derivative of $(\delta_e)_{pull\text{-}up}$ with 'n' is called elevator angle per g and from Eq.(4.9) it is given by:

$$\left(\frac{d\delta_e}{dn} \right)_{pull\text{-}up} = \frac{1}{V^2} \left\{ -\frac{2W}{\rho S} \frac{(dC_m / dC_L)_{stick\text{-}fixed}}{C_{m\delta}} - \frac{63 \, g l_t}{\tau} \right\} \tag{4.10}$$

Remark:

In level flight, $(d\delta_e /dC_L)$ is zero when $(dC_m/dC_L)_{stick\text{-}fixed}$ is zero. From Eq.(4.10) it is seen that $(d\delta_e /dn)$ is not zero when $(dC_m/dC_L)_{stick\text{-}fixed}$ is zero. This is because the damping produced in a pull-out makes the airplane apparently more stable. From Eq.(4.10) $(d\delta_e /dn)$ is zero when $(dC_m/dC_L)_{stick\text{-}fixed}$ has the following value:

$$\left(\frac{dC_m}{dC_L} \right)_{stick\text{-}fixed} = - \frac{63 \, g \, l_t \, \rho \, C_{m\delta}}{2 \tau (W / S)} \tag{4.11}$$

4.4 Stick-fixed maneuver point (x_{mp})

The c.g. location for which $(d\delta_e/dn)_{pull\text{-}up}$ is zero is called stick-fixed maneuver point and denoted by (x_{mp}). From Eq.(4.11) and noting that $(dC_m/dC_L)_{stick\text{-}fixed}$ is zero when c.g. is at x_{NP}, the following expression is obtained for x_{mp}.

$$\frac{x_{mp}}{\bar{c}} = \frac{x_{NP}}{\bar{c}} - \frac{63 \ g \ l_t \ \rho \ C_{m\delta}}{2\,\tau\left(\dfrac{W}{S}\right)} \tag{4.12}$$

Using Eq(4.12) in (4.10) gives :

$$\left(\frac{d\delta_e}{dn}\right)_{pull-up} = -\frac{C_L}{C_{m\delta}}\left\{\frac{x_{cg}}{\bar{c}} - \frac{x_{mp}}{\bar{c}}\right\} \tag{4.12a}$$

Example 4.1

Consider an airplane with W = 22500 N, S = 15 m^2, aspect ratio = 6, $\bar{c}$ = 2.50 m, $l_t = 3\bar{c}$ = 7.5m, $C_{m\delta}$ = -0.01 deg^{-1} and τ = 0.5. Calculate the difference between maneuver point stick-fixed and neutral point stick-fixed.

Solution:

Substituting the given values in Eq.(4.12) and assuming sea level conditions i.e. ρ = 1.225 kg/m^3 gives:

$$\frac{x_{mp}}{\bar{c}} - \frac{x_{NP}}{\bar{c}} = \frac{- \ 63 \times 9.81 \times 7.5 \times 1.225 \times (-0.01)}{2 \times 0.5 (22500/15)} = 0.0378$$

4.5 Stick force gradient in pull-up

The stick force (F) is given by :

$$F = G \ \frac{1}{2} \ \rho \ V^2 \ \eta \ \bar{c}_e \ S_e \{C_{h\alpha t} \ \alpha_t + C_{h\delta e} \ \delta_e + C_{h\delta t} \ \delta_t\}$$

Substituting for α_t and δ_e yields:

$$F = G \ \frac{1}{2} \ \rho \ V^2 \ \eta \ \bar{c}_e \ S_e \ [C_{h\alpha t} \ (\alpha_{0Lw} + i_t - i_w) + C_{h\delta e} \ \delta_{e0CL} + C_{h\delta t} \ \delta_t$$

$$-\frac{2n(W/S)}{\rho V^2 C_{m\delta}} C_{h\delta e} \left(\frac{dC_m}{dC_L}\right)_{stick-free} + \frac{57.3 \ g \ l_t \ (n-1)}{V^2}\{C_{h\alpha t} - 1.1 \ \frac{C_{h\delta e}}{\tau}\}] \tag{4.13}$$

Hence,

$$\left(\frac{dF}{dn}\right)_{pull-up} = -\frac{G\eta \ \bar{c}_e \ S_e \left(\dfrac{W}{S}\right) C_{h\delta e}}{C_{m\delta}}\left(\frac{dC_m}{dC_L}\right)_{stick-free} + 57.3 G\eta S_e \bar{c}_e g l_t \frac{\rho}{2}\{C_{h\alpha t} - 1.1\frac{C_{h\delta e}}{\tau}\} \tag{4.14}$$

The quantity dF/dn is called the stick force gradient per g.

4.6 Maneuver point stick-free

The c.g. location for which (dF/dn) equals zero is called maneuver point stick -free. It is denoted by x'_{mp} . Recalling that the stick-free neutral point is denoted by x'_{NP}, the following expression is obtained for x'_{mp} :

$$\frac{x'_{mp}}{\bar{c}} = \frac{x'_{NP}}{\bar{c}} + \frac{57.3\ g\ l_t\ \rho\ C_{m\delta}}{2(\frac{W}{S})C_{h\delta e}} \{C_{h\alpha t} - 1.1\ \frac{C_{h\delta e}}{\tau}\} \tag{4.15}$$

Using Eq(4.15) in Eq(4.14) gives $\left(\dfrac{dF}{dn}\right)_{pull-up}$ stick-force gradient per g as :

$$\left(\frac{dF}{dn}\right)_{pull-up} = -\ G\ \bar{c}_e\ S_e\ \eta\ (W/S)\frac{C_{h\delta e}}{C_{m\delta}}\left\{\frac{x_{cg}}{\bar{c}} - \frac{x'_{mp}}{\bar{c}}\right\} \tag{4.16}$$

Example 4.2

For the airplane in example 4.1 assume further that $C_{h\alpha t}$ = - 0.003 deg^{-1} and $C_{h\delta e}$ = - 0.005 deg^{-1}. Obtain the difference between stick-free maneuver point and stick-free neutral point.

Substituting various quantities in Eq.(4.15) gives:

$$\frac{x'_{mp}}{\bar{c}} - \frac{x'_{NP}}{\bar{c}} = \frac{57.3 \times 9.81 \times 7.5 \times 1.225 \times (-0.01)}{2 \times 1500\ (-0.005)} \{(-\ 0.003) - \frac{1.1 \times (-0.005)}{0.5}\} = 0.0275$$

$$\frac{x'_{mp}}{\bar{c}} = \frac{x'_{NP}}{\bar{c}} + 0.0275$$

Remark:

x'_{mp} lies behind x'_{NP} because the airplane has acquired apparent increase in stability in a pull up.

4.7 Limits on stick force gradient per g

The stick force gradient per g or (dF/dn) indicates ease or difficulty in carrying out a maneuver. Hence it should lie within certain limits. Reference 1.7 gives the limits as 3 lbs/g to 8 lbs/g or 14 N/g to 36 N/g for fighters and the upper limit of 156 N/g for bombers and cargo airplanes. As (dF/dn) depends on c.g. location, these limits impose restrictions on c.g. travel (see section 4.9).

4.8 Static stability and control in a turn

References 1.7 and 1.12 consider, in addition to pull up, the stability and control in a turning flight. However, the requirements in this flight are less critical than those in a pull up.

4.9 Overall limits on c.g. travel

Taking into account various considerations discussed in chapters 2, 3 and 4, the limits on c.g. travel are shown in the Fig.4.4. It is to be noted that generally, c.g. travel should be limited to about 8% of m.a.c. for a general aviation airplane and about 15% of m.a.c. for a passenger airplane. The stringent nature of limitations on c.g. travel can be gauged from the following case. For an airplane of length =10 m, b = 10 m and aspect ratio = 10, the value of $\bar{c}$ would be around one metre. Thus, a permissible c.g. travel of 8% implies just 8 cm of c.g movement for a fuselage of 10 m length.
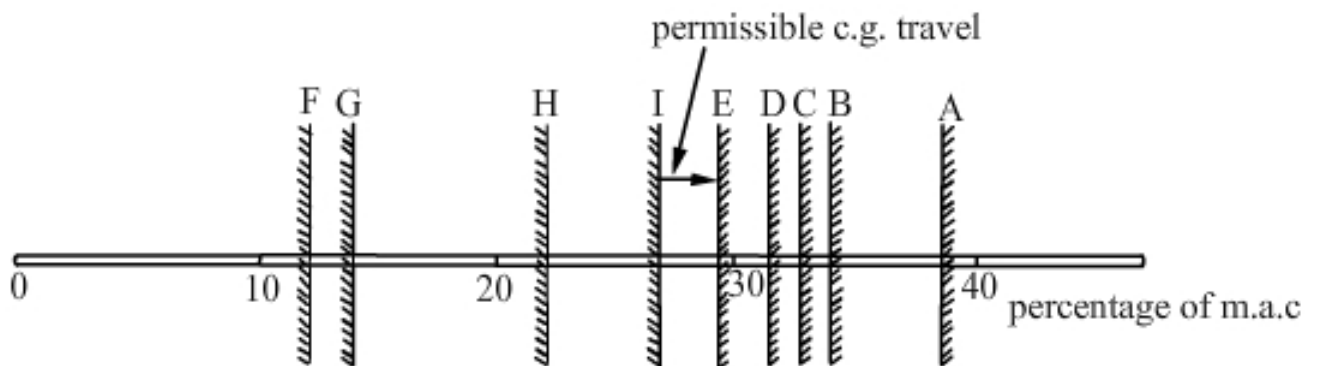
Note: The symbols A,B… , I indicate limitations on c.g. movement due to the following considerations.

Limits on aft c.g. movement: A: $(x_{NP})_{power\ off}$; B: $(dF/dn) = 0$; C: $(x_{NP})_{poweron}$;

D: $(dF/dn)_{minimum}$; E: $(x'_{NP})_{poweron}$.

Limits on forward c.g. movement:

F: δ_e for C_{Lmax} in free flight with n=1; G:δ_e for C_{Lmax} in free flight with n = n_{max};

H: δ_e for C_{Lmax} with ground effect; I: $(dF/dn)_{max}$.

Fig.4.4 Summary of limits on c.g. travel due to various considerations- schematic

4.10 Remark on determination of x_{mp} and x'_{mp} from flight tests

In sections 2.13 and 3.5 the flight test techniques for determination of x_{NP} and x'_{NP} are mentioned. The procedure to determine x_{mp} and x'_{mp} can be briefly described as follows.

(i) Choose a c.g. location,

(ii) Choose a flight speed,

(iii) Fly the airplane in a pull-up or a steady turn at chosen load factor (n). During the test, the values of average altitude, flight speed, elevator deflection, control force and load factor are noted. Corrections to the readings are applied for any errors. Carry out the tests at different flight speeds and load factors.

(iv) Repeat tests at different c.g. locations.

(v) From the flight tests for a chosen c.g. location but at different values of n, plot δ_e vs n and F vs n. Calculate $d\delta_e/dn$ and dF/dn.

(vi) Obtain $d\delta_e/dn$ and dF/dn at different c.g. locations.

(vii) Plot ($d\delta_e/dn$) vs c.g. location. Extrapolate the curve. The c.g. location for which ($d\delta_e/dn$) equals zero is the maneuver point stick-fixed (x_{mp}).

(viii) Plot dF/dn vs c.g. location. Extrapolate the curve. The c.g. location for which (dF/dn) equals zero is the maneuver point stick-free (x'_{mp}). For further details, see chapter 4 of Ref.2.5.

Example 4.3

Given an airplane with the following geometric and aerodynamic characteristics:

S = 19.8 m^2, b = 10.5 m, $C_{L\alpha w}$ = 0.078 deg^{-1}, $\bar{c}$ = 2.2 m; l_t = 5.0 m, i_w = 2^0, $d\varepsilon$ / $d\alpha$ = 0.48, S_t = 3.6 m^2, span of tail plane (b_t) = 4.0 m, $C_{L\alpha t}$ = 0.058 deg^{-1}, i_t = 1^0, S_e = 1.08 m^2, $\bar{c}_e$ = 0.28 m; η = 0.9; $C_{h\alpha t}$ = -0.004 deg^{-1}, $C_{h\delta e}$ = -0.009 deg^{-1}, G = 1.6 m^{-1}, W = 40,000 N,   (x_{NP}) = 0.35$\bar{c}$. Calculate stick force gradient in pull-up at sea level for c.g. location of 0.20$\bar{c}$, 0.26$\bar{c}$ and 0.37$\bar{c}$. Plot these values versus c.g. position and by graphical interpolation or extrapolation

determine stick-free maneuver point. If it is required that the airplane has stick force per g between 36 N/g and 14 N/g . What would be the c.g. limits? Assume that the tab is always used to trim out stick force at n =1.

Solution:

i) Calculation of $(dF/dn)_{pull-up}$.

$$V_H = \frac{S_t}{S}\frac{l_t}{\bar{c}} = \frac{3.6}{19.8}\frac{5.0}{2.2} = 0.4132$$

$$(\frac{dF}{dn})_{pull-up} = -\frac{G\,\eta\,S_e\,\bar{c}_e\,(W/S)}{C_{m\delta}}\,C_{h\delta}\,(\frac{dC_m}{dC_L})_{stick-free} + 57.3G\,\eta\,S_e\,\bar{c}_e\,g\,\,l_t\frac{\rho}{2}(C_{h\alpha}-\frac{1.1C_{h\delta}}{\tau})$$

$S_e/S_t = 1.08/3.6 = 0.3$; hence, $\tau = 0.5$ (Fig. 2.32)

$C_{m\delta} = -V_H\eta\,C_{L\alpha t}\,\tau = -0.4132\times0.9 \times 0.058 \times 0.5 = -0.0108$ deg^{-1}

$W/S = 40000/19.8 = 2020$ N/m^2

$x_{NP} = 0.35\bar{c}$

$$x'_{NP} = x_{NP} - V_H\eta\frac{C_{L\alpha t}}{C_{L\alpha w}}(1-\frac{d\varepsilon}{d\alpha})\tau\frac{C_{h\alpha t}}{C_{h\delta e}}$$

$$\frac{x'_{NP}}{\bar{c}} = 0.35-0.4132\times0.9 \times \frac{0.058}{0.078}\times(1-0.48)\times0.5\times\frac{(-0.004)}{(-0.009)} = 0.35-0.032 = 0.318$$

$$\frac{G\,\eta S_e\bar{c}_e(W/S)C_{h\delta}}{C_{m\delta}} = \frac{1.6\times0.9\times1.08\times0.28\times2020\times(-0.009)}{(-0.0108)} = -733.02$$

$$57.3\times G\,\eta\,S_e\,\bar{c}_e\,g\,\,l_t\frac{\rho}{2}(C_{h\alpha}-\frac{1.1C_{h\delta}}{\tau})$$

$$= 57.3\times1.6\times0.9\times1.08\times0.28\times9.81\times5.0\times\frac{1.225}{2}\{-0.004-1.1\times\frac{(-0.009)}{0.5}\} = 11.85$$

$$(\frac{dF}{dn})_{pull-up} = -733.02\,(dC_m/dC_L)_{stick-free} +11.85 \qquad\qquad (E\ 4.3.1)$$

$(dC_m/dC_L)_{stick-free}$ for different location of c.g. are given in table E4.3. Further using Eq. (E4.3.1), the values of $(dF/dn)_{pull-up}$ are also given in the same table.

c.g.	$(dC_m/dC_L)_{stick-free}$	$(dF/dn)_{pull-up}$
$0.2\bar{c}$	-0.118	98.34
$0.26\bar{c}$	-0.058	48.66
$0.37\bar{c}$	0.052	-26.25

Table E4.3 $(dC_m/dC_L)_{stick-free}$ and $(dF/dn)_{pull-up}$ for different locations of c.g.

(ii) Maneuver point stick-free : This is the c.g. location for which $(dF/dn)_{pull-up}$ is zero. From Eq.(E4.3.1):

$0 = -733.02(dC_m / dC_L)_{stick-free} + 11.85$

or $(dC_m/dC_L)_{stick-free}$ at maneuvre point stick-free $= 11.85 / 733.02 = 0.016$

Hence, maneuvre point stick free is:

$= (0.318 + 0.016) \bar{c} = 0.334 \bar{c}$

(iii) c.g. limits for dF/dn between 14 to 36 N/g can be obtaining using corresponding $(dC_m/dC_L)_{stick-free}$.

$14 = -733.02 (dC_m/dC_L)_{stick-free1} + 11.85$

$(dC_m/dC_L)_{stick-free1} = -2.15 / 733.02 = -0.00293$

$36 = -733.02(dC_m/dC_L)_{stick-free2} + 11.85$

$(dC_m/dC_L)_{stick-free2} = \dfrac{-(36 - 11.85)}{733.02} = -0.0329$

Hence, c.g. limits are $(0.318-0.0329) \bar{c}$ & $(0.318-0.0029)\bar{c}$ i.e. $0.285 \bar{c}$ & $0.315 \bar{c}$.

Remark:

In this particular example, the c.g. limits for proper dF/dn seem to be too narrow; the limits on (dF/dn) perhaps correspond to a fighter airplane.