Chapter 5

Directional static stability and control - 2

Lecture 17

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5.5 Contribution of power to $C_{n\beta}$

Figure 5.5 shows a tractor propeller in sideslip. It produces a side force Y_p and yawing moment $Y_p l_p$. Since, the moment depends on angle of sideslip, there is a direct contribution to $C_{n\beta}$. It is denoted by $C_{n\beta p}$. It is seen that the contribution is negative and hence, destabilizing. If the airplane has a pusher propeller, then the contribution is positive and stabilizing. A jet engine in sideslip will also produce $C_{n\beta p}$ whose value will depend on the engine location.

A propeller also has an indirect contribution to $C_{n\beta}$. The slipstream of a propeller in sideslip would be asymmetric (Fig.5.6). It is observed that for a positive value of β, the left wing will have a larger region influenced by the propeller slipstream than the right wing. Since, the dynamic pressure in the slipstream is higher than the free stream dynamic pressure, the left wing, with larger region influenced by slip stream, will have higher drag than the right wing. This would result a slight destabilizing contribution to $C_{n\beta}$.

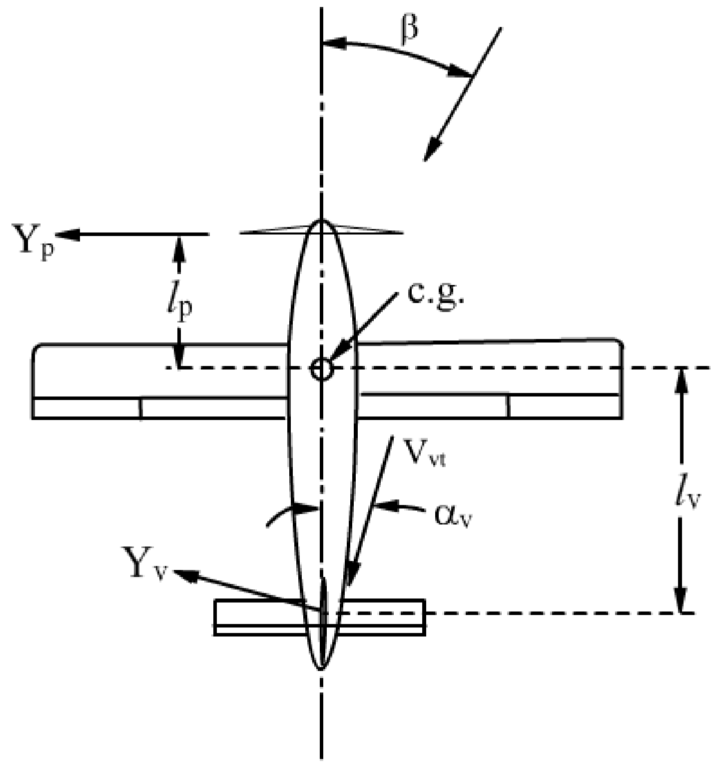

Fig.5.5 Propeller and vertical tail in sideslip

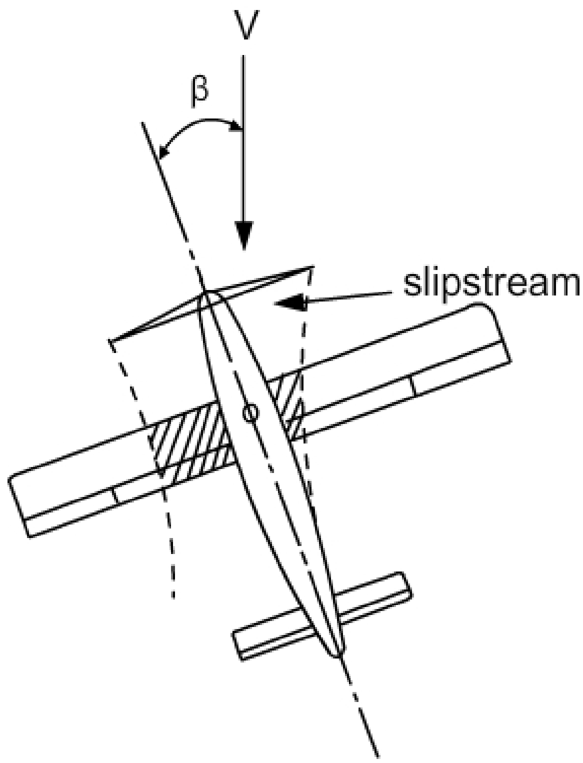

Fig.5.6 Slip stream of a propeller in sideslip

Remark:

An accurate estimate of $C_{n\beta p}$ is difficult due to the influence of various factors.It is generally small and ignored during initial estimate of $C_{n\beta}$.

5.6 Contribution of vertical tail

In subsection 2.4.4. it is shown that the horizontal tail at an angle of attack produces lift L_t and a pitching moment M_{cgt}. Similarly, a vertical tail at an angle of attack (α_v) would produce a side force (Y_v) and a yawing moment (N_v) (See Fig.5.5).The side force Y_v is perpendicular to the velocity V_{vt} as shown in Fig.5.5. However, the angle α_v is small and Y_v is taken perpendicular to FRL.

Now,

$$Y_v = -C_{L\alpha v}\,\alpha_v \frac{1}{2}\rho V_{vt}^2 S_v \tag{5.13}$$

Note that as per convention Y_v is positive in the direction of y-axis. Hence, positive β gives negative Y_v. The yawing moment due to vertical tail is given as:

$$N_v = C_{L\alpha v}\,\alpha_v \frac{1}{2}\rho V_{vt}^2 S_v\, l_v \tag{5.14}$$

and $$C_{nv} = \frac{C_{L\alpha v}\,\alpha_v \frac{1}{2}\rho V_{vt}^2 S_v l_v}{\frac{1}{2}\rho V^2 S b} \qquad (5.14a)$$

5.6.1 Influence of wing-body combination on contribution of vertical tail

The wing body combination has the following influences.

(a) The angle of attack (α_v) at vertical tail is different from β and (b) The dynamic pressure (½ ρV^2_{vt}) experienced by it (vertical tail) is different from (½ ρV^2) (Figs.5.5, 5.7). The angle of attack is modified as:

$$\alpha_v = \beta + \sigma\,; \qquad (5.15)$$

where, σ is called side wash.

The dynamic pressure experienced by tail is expressed as:

$$\tfrac{1}{2}\rho V^2_{vt} = \eta_v\,(\tfrac{1}{2}\rho V^2) \qquad (5.16)$$

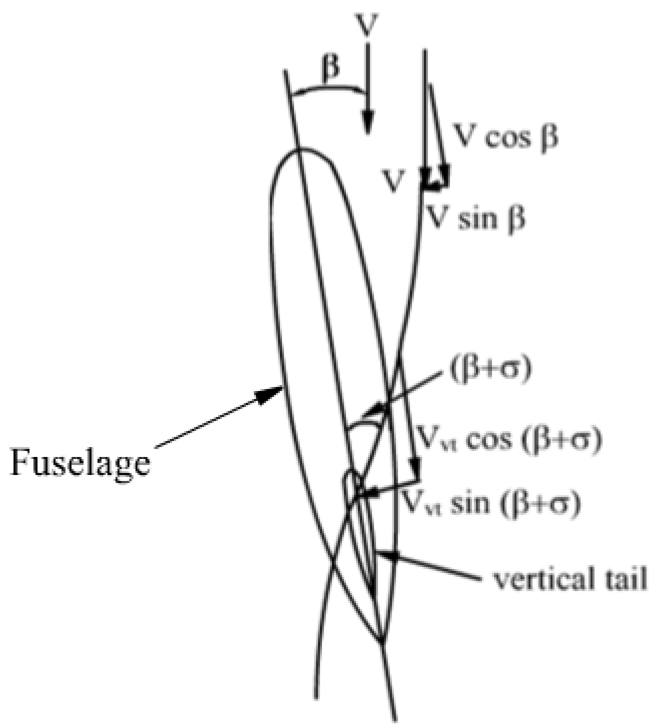

Fig.5.7 Side wash on vertical tail

5.6.2 Expression for $C_{n\beta V}$

Taking into account the interference effects Eq.(5.14a) becomes:

$$C_{nv} = \frac{C_{L\alpha v}\ (\beta+\sigma)\frac{1}{2}\ \rho\ V_{vt}^2\ S_v}{\frac{1}{2}\rho\ V^2\ Sb}$$

$$= V_v\, \eta_v\ C_{L\alpha v}\,(\beta+\sigma) \tag{5.17}$$

where,

$$V_v = \frac{S_v}{S}\frac{l_v}{b} \text{ and } \eta_v = \frac{\frac{1}{2}\ \rho\ V_{vt}^2}{\frac{1}{2}\rho V^2}; \tag{5.18}$$

Differentiating Eq.(5.17) by β gives :

$$C_{n\beta v} = V_v\ \eta_v\ C_{L\alpha v}\ (1+\frac{d\sigma}{d\beta}) \tag{5.19}$$

As mentioned earlier, the wing and fuselage influence σ and η_V (Fig.5.7). Based on Ref.2.2, the following empirical formula gives the influence of wing-body combination.

$$\eta_v(1+\frac{d\sigma}{d\beta}) = 0.724 + 3.06\ \frac{S_v/S}{1+\cos\Lambda_{c/4w}}\ + 0.4\ \frac{z_w}{d}\ + 0.009\ A_w \tag{5.20}$$

where z_W is the distance, parallel to z-axis , between wing root quarter chord point and the FRL ; d is the maximum depth of the fuselage; and $\cos\Lambda_{c/4w}$ is sweep of wing quarter chord line.

Remark:

The slope of lift curve of the vertical tail ($C_{L\alpha V}$) can be calculated by knowing its effective aspect ratio (A_{veff}) and using methods similar to those for estimation of the slope of lift curve of the wing. The aspect ratio of the vertical tail (A_v) is b_V^2 / S_V, where b_v is generally the height of vertical tail above the centre line of the portion of the fuselage where the vertical tail is located and S_v is the area of the vertical tail above the aforesaid centre line. The effective aspect ratio

(A_{veff}) of the vertical tail is much higher than A_v and depends on the interference effect due to fuselage and horizontal tail. For details see appendix 'C', and Ref.1.8b, Ref.1.12 , chapter 3 and Ref. 1.7, chapter 8.

5.7 Directional static stability

Having obtained the contributions of various components to $C_{n\beta}$, various aspects of directional static stability are discussed below.

5.7.1 Pedal-fixed static stability

As regards the action of pilot to effect the movement of control surfaces, the following may be pointed out.

(a) The elevator is moved by the forward or backward movement of the control stick. (b) The ailerons are operated by the sideward movement of the control stick. (c) The rudder is moved by pushing the pedals.

In longitudinal static stability analysis the stick-fixed and stick-free cases were considered. These deal with the elevator-fixed and elevator-free cases respectively. Similarly, in directional static stability, rudder-fixed and rudder-free stability is considered. These cases are also referred to as pedal-fixed and pedal-free stability analyses.

In the analysis of directional static stability carried out so far the contributions of wing, fuselage, nacelle, power and vertical tail to $C_{n\beta}$ have been considered. Noting that for the pedal-fixed stability, the rudder deflection is constant, $(C_{n\beta})_{pedal\text{-}fixed}$ is given by adding these individual contributions i.e.

$$C_{n\beta} = (C_{n\beta})_w + (C_{n\beta})_{f,n,p} + V_V\ \eta_v\ C_{L\alpha v}\ (1+\frac{d\sigma}{d\beta}) \qquad (5.21)$$

5.7.2 Weathercock effect

Whenever an airplane, originally flying with zero sideslip, develops a sideslip (β), the vertical tail tends to bring it back to the original position of zero sideslip. This effect is similar to that of the vane attached to the weathercock which is used to indicate the direction of wind and is located on top of buildings in meteorological departments and near airports (Fig.5.8). When the vane is at an angle of attack, it produces lift on itself and consequently a moment about its hinge. This moment becomes zero only when the vane is aligned with the wind direction. Hence, the vane is always directed in a way that the arrow points in the direction opposite to that of the wind. The action of vertical tail on the airplane is also similar to that of the vane and helps in aligning the airplane axis with wind direction. Hence, the directional stability is also called weathercock stability.

Fig.5.8 Weathercock or weather vane

(Source:www.pro.corbis.com)

5.7.3 Pedal-free static directional stability:

As noted in section 3.1 the hinge moment on the elevator is made zero by suitable deflection of the elevator tab. Similarly, the hinge moment of the rudder is also brought to zero by suitable deflection of the rudder tab. The analysis of static stability when rudder is left free to move is called rudder-free or pedal-free stability.

The equation for hinge moment about rudder hinge (C_{hr}) can be expressed as:

$$C_{hr} = C_{h\alpha v}\alpha_v + C_{h\delta r}\delta_r + C_{h\delta rtab}\,\delta_{rt} \tag{5.22}$$

Where, δ_r is the rudder deflection and δ_{rt} is the deflection of the rudder tab. The floating angle of rudder, δ_{rfree} is obtained when C_{hr} is zero i.e.

$$\delta_{r\,free} = -\frac{(C_{h\alpha v}\,\alpha_v + C_{h\delta rtab}\,\delta_{rt})}{C_{h\delta r}} \tag{5.23}$$

Hence, $$\frac{d\delta_{r\,free}}{d\beta} = -\frac{C_{h\alpha v}}{C_{h\delta r}}\frac{d\alpha_v}{d\beta} = -\frac{C_{h\alpha v}}{C_{h\delta r}}(1+\frac{d\sigma}{d\beta}) \tag{5.24}$$

Remarks:

i) Convention for rudder deflection

A rudder deflection to left is taken as positive. It is a general convention that a positive control deflection produces negative moment. This is consistent with the convention that rotation is taken positive clockwise when looking along the axis about which the rotation takes place.

ii) $C_{h\alpha v}$ and $C_{h\delta r}$ and level of static direction stability with rudder free

Noting the convention for β, and with the help of pressure distribution shown in Fig.3.3, it is observed that a positive value of β would produce a negative force on the rudder and hence a positive hinge moment. Consequently, $C_{h\alpha v}$ is positive. In a similar manner it is observed that a positive rudder deflection would produce a positive side force and hence, negative hinge moment. Thus, $C_{h\delta r}$ is negative. Hence, from Eq.(5.24), $(d\delta_r)_{free} / d\beta$ is negative. Thus, when a disturbance produces positive β, the rudder will take such a position that the rudder deflection is negative. The result is a negative change in yawing moment or reduced static stability. Thus, the level of static directional stability will be reduced when the pedal is free.

5.7.4 Desirable level of $C_{n\beta}$

In longitudinal static stability, the shift of c.g. has a profound effect on the level of stability ($C_{m\alpha}$) as the contribution of wing to $C_{m\alpha}$ depends directly on $(x_{cg} - x_{ac})$. Thus, the shift in the position of c.g, during flight, almost decides the area of the horizontal tail. However, a shift of c.g. does not cause a significant change in $C_{n\beta}$ because such a change may only have a secondary effect by way of slightly affecting l_v. Hence, to arrive at the area of the vertical tail, a criterion to prescribe a desirable value of $C_{n\beta}$ is needed. Reference 1.7, Chapter 8 gives:

$$(C_{n\beta})_{desirable} = 0.005\,(\frac{W}{b^2})^{1/2}\ deg^{-1} \tag{5.25}$$

where, W is the weight of the airplane in lbs and b is the wing span in feet.

Reference 1.1, Appendix 'B' gives characteristics of seven airplanes. It is observed that for the subsonic airplanes $C_{n\beta}$ lies between 0.0013 to 0.0026 deg^{-1}. However, the final value of $C_{n\beta}$ is decided after the dynamic stability analysis. From lateral dynamic stability analysis (chapter 9) it will be observed that a large value of $C_{n\beta}$ leads to some unacceptable response of the airplane to the disturbance.

Example 5.2

A model of an airplane is tested in a wind tunnel without the vertical tail. Contributions of various components give $C_{n\beta}$ = -0.0012 deg^{-1}. If the vertical tail is to be positioned at a point on the aft end of the fuselage giving a tail length of 4.8 m, How much vertical tail area is required to give an overall $C_{\eta\beta}$ = 0.0012 deg^{-1}? Assume that the vertical tail would have an effective aspect ratio of 2, the wing area is 18 m^2, wing span is 10.6 m and the wing is set at the middle of the fuselage.

Solution:

$C_{n\beta} = (C_{n\beta})_w + (C_{n\beta})_{f,n,p} + (C_{n\beta})_{vt}$

Given: $(C_{n\beta})_w + (C_{n\beta})_{f,n,p} = -0.0012$

$(C_{n\beta})_{required} = 0.0012$

Hence, $(C_{\eta\beta})_{vt} = 0.0012 - (- 0.0012) = 0.0024$

From Eq.(5.19),

$$(C_{n\beta})_v = V_v \eta_v C_{L\alpha v} (1+\frac{d\sigma}{d\beta})$$

I) Estimation of $C_{L\alpha v}$:

$A_{veff} = 2$.

The expression for $C_{L\alpha v}$ is (Ref.1.8b):

$$C_{L\alpha} = \frac{2\pi A}{2+\sqrt{\frac{A^2\beta_M^2}{K^2}\frac{(1+\tan^2\Lambda_{c/2})}{\beta^2}+4}} \text{ in rad}^{-1}$$

where, $\beta_M = (1-M^2)^{1/2}$, K = lift curve slope of airfoil / 2π.

$\Lambda_{c/2}$ = sweep of mid chord line

As Mach number is low subsonic $\beta_M \approx 1$. Let K =1, $\Lambda_{\frac{c}{4}w} = 0$

Consequently,

$$C_{L\alpha} = \frac{2\pi A}{2+\sqrt{A^2+4}}$$

For A_{Veff} of 2.0, $C_{L\alpha v} = \dfrac{2\pi\times 2}{2+\sqrt{2^2+4}} = 2.60\ \text{rad}^{-1} = 0.0454\ \text{deg}^{-1}$

$$A_w = \frac{10.6^2}{18} = 6.24$$

Remark:

Reference 1.7, Fig.8.8 gives $C_{L\alpha v}$=0.044 deg^{-1} for A_{veff} of 2.0.

II) Estimation of $\eta_v(1+\dfrac{d\sigma}{d\beta})$

The expression for $\eta_v(1+\dfrac{d\sigma}{d\beta})$ as given by Eq.(5.20) depends on S_v / S but S_v / S is not known at this stage. Hence, as a first approximation it is assumed that $S_v / S = 0.12$. The quantity z_w/d can be taken as zero for the mid wing configuration.

$$\text{Hence}, \eta_v(1+\frac{d\sigma}{d\beta}) = 0.724 + 3.06(\frac{0.12}{1+1}) + 0 + 0.009 \times 6.24 = 0.964$$

Consequently, the first estimation of V_v is:

$0.0024 = V_v \times 1 \times 0.0454 \times 0.964$ or $V_v = 0.05484$

Noting that, $V_v = \dfrac{S_v}{S}\dfrac{l_v}{b}$, gives $S_v = 0.05484 \times 18 \times \dfrac{10.6}{4.8} = 2.18\,m^2$

To improve the estimation of S_v,its value in the previous step is substituted in the expression for $\eta_v(1+\dfrac{d\sigma}{d\beta})$i.e.

$$\eta_v(1+\frac{d\sigma}{d\beta}) = 0.724+3.06(\frac{2.18/18}{1+1})+0+0.009\times 6.24 = 0.9655$$

The second estimation of V_v is:

$$V_v = \frac{0.0024}{0.9655 \times 0.0454} = 0.05475$$

Or $S_v = 2.176\ m^2$

Since, the two estimates are close to each other, the iteration is terminated and $S_v = 2.176\ m^2$ is taken as the answer.

5.8 Directional Control

Control of rotation of the airplane about the z-axis is provided by the rudder.

The critical conditions for design of rudder are:

(a) Adverse yaw,

(b) Cross wind take-off and landing,

(c) Asymmetric power for multi- engined airplanes and

(d) Spin

5.8.1 Adverse yaw and its control

When an airplane is rolled to the right, the rate of roll produces a yawing moment tending to turn the airplane to the left. Similarly, a roll to left produces yaw to right. Hence, the yawing moment produced as a result of the rate of roll is called adverse yaw. To explain the production of adverse yaw, consider an airplane rolled to right, i.e. right wing down. Let, the rate of roll be ‘p’. The rate of roll produces the following two effects.

a) A roll to right implies less lift on the right wing and more lift on the left wing. This is brought about by aileron deflection – in the present case an up aileron on the right wing and a down aileron on the left wing. Since, C_L on the right wing is less than C_L on the left wing, the induced drag coefficient (C_{Di}) on the right wing is less than C_{Di} on left wing. This results in a yawing moment causing the airplane to yaw to left.

b) Due to the rolling velocity (p) a section on the down going wing at a distance y from the FRL experiences a relative upward wind of magnitude ‘py’. At the same time a section on the up going wing at a distance y from FRL experiences a relative downward velocity of magnitude ‘py’. This results in the change of direction of the resultant velocity on the two wing halves (Fig.5.9). Now, the lift vector, being perpendicular to the resultant velocity, is bent forward on the down going wing and bent backwards on the up going wing.Consequently, the horizontal components of the lift on the two wing halves produce a moment tending to yaw the airplane to left. An approximate estimate of the effect of adverse yaw is (Ref.1.1, chapter 3):

$$(C_n)_{adverseyaw} \approx -\frac{C_L}{8}\frac{pb}{2V} \qquad (5.27)$$

where, p = rate of roll in radians per second; b = wing span and V = flight velocity .

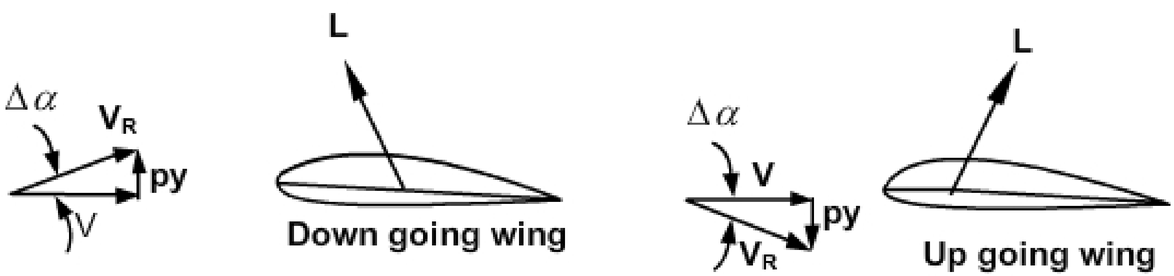

Fig.5.9 Effect of rate of roll

An airplane is generally designed for a specific value of (pb/2V). For example, Ref.1.7 Chapter 9 prescribes that up to 80% of V_{max} the airplane should have:

pb / 2V = 0.07 for cargo/ bomber

pb / 2V = 0.09 for fighter

Hence, one of the criteria for rudder design is that it must be powerful enough to counter the adverse yaw at prescribed rate of roll.

5.8.2 Control in cross wind take-off and landing

An Airplane sometimes encounters side winds during take-off and landing. As regards control during this eventuality, the following three points may be noted.

(1) When an airplane flying at a velocity 'V', encounters a side wind of velocity 'v', the resultant velocity vector makes an angle $\Delta\beta$ to the plane of symmetry;

$\Delta\beta = v/V$.

(2)The tendency of an airplane possessing directional static stability, is to align itself with the wind direction (weather cock effect).

(3) During take-off and landing the pilot has to keep the airplane along the runway. Hence, when a cross wind is present the airplane is side-slipping with angle $\Delta\beta$.

Thus, another criterion for the design of the rudder is required. It must be able to counteract the yawing moment due to sideslip produced by the cross wind ($C_{n\beta}$ x $\Delta\beta$). This criterion becomes more critical at lower speeds because (a) the effectiveness of the rudder, being proportional to V^2, is less at lower flight speeds and (b) $\Delta\beta$ being proportional to 1/V, is high at low flight speeds.

According to Ref.1.1, chapter 2 the rudder must be able to overcome v = 51 ft / s or 15 m/s at the minimum speed for the airplane. It may be pointed out that on a rainy day, with heavy cross winds, the landing on the airport may be refused if the cross wind is more than that permitted for the airplane.