# Chapter 1

# Lecture 2

# Introduction - 2

## Topics



### 1.3.3 Static stability and dynamic stability

In the cases illustrated by Fig.1.6 a, b, c and d, it is observed that, as soon as the the system is disturbed, it tends to return to the undisturbed position. Such systems are called statically stable.

For the case in Fig.1.6e, the tendency of the system, immediately after the disturbance, is to turn away from the equilibrium position. Such a system is said to be statically unstable.

When the tendency of the system, after the disturbance, is to stay in the disturbed position, then it is said to have neutral static stability.

Even when the system has a tendency to go towards the undisturbed position (cases 1.6a, b, c and d), it may not return to the equilibrium position as in the cases shown in Fig.1.6 b & c namely divergent oscillation and undamped oscillation. Only when the system finally returns to the equilibrium position, the system is said to be dynamically stable. Otherwise, it is dynamically unstable. With this criterion, the damped oscillation and subsidence are the only dynamically stable cases.

**Remarks:**

i) The definitions of the terms static stability and dynamic stability are as follows:

**Static Stability:** A system is said to be statically stable when a small disturbance causes forces and moments that tend to move the system towards its undisturbed position. If the forces and moments tend to move the system away from the equilibrium position, then the system is said to be statically unstable. In the case of a system having neutral static stability, no forces or moments are created as a result of the disturbance.

**Dynamic Stability:** A system is said to be dynamically stable if it eventually returns to the original equilibrium position after being disturbed by a small disturbance.

ii) It is obvious from the above discussion that for a system to be dynamically stable, it must be statically stable. Table 1.1 categories the cases in Fig.1.6 as regards the static stability and dynamic stability.

| Case | Figure | Static stability | Dynamic stability |
|---|---|---|---|
| Damped oscillation | 1.6a | Yes | Yes |
| Divergent oscillation | 1.6b | Yes | No |
| Undamped oscillation | 1.6c | Yes | No |
| Subsidence | 1.6d | Yes | Yes |
| Divergence | 1.6e | No | No |
| Neutral stability | 1.6f | No | No |

Table 1.1 Static and dynamic stability

iii) The distinction between static stability and dynamic stability is of special significance in aeronautical applications as the analysis of static stability is much simpler than that of the dynamic stability. This can be explained as follows.

The disturbance to an airplane in flight due to a gust may change its angle of attack ($\alpha$) or sideslip ($\beta$) or bank ($\varphi$) or the thrust output. Now, these changes may produce changes in aerodynamic forces and moments. If these forces and moments tend to bring the airplane to the original state, then the airplane is statically stable. Thus, to assess the static stability, one needs only to examine the aerodynamic / propulsive forces and moments brought about at the time the disturbance is applied. On the other hand, to examine the dynamic stability of the airplane, one has to consider the subsequent motion which involves accelerations and hence, the inertia forces. Further, the dynamic stability analysis requires solution of the equations of motion taking into account the changes, with time, in aerodynamic forces and moments due to changes in $\alpha$,$\beta$,

$\varphi$, and the linear and angular velocities etc. of the airplane. These quantities denoting changes in aerodynamic forces and moments due to aforesaid changes are called aerodynamic / stability derivatives. Hence, in aeronautical engineering practice first the static stability is ensured by providing adequate areas of horizontal tail and vertical tail and the dihedral angle. Subsequently, the dynamic stability analysis is carried out to ensure that there is adequate damping.

### 1.3.4 Recapitulation of some terms – body axes system, earth fixed axes system, attitude, angle of attack and angle of sideslip

At this stage a brief discussion on body axes system, attitude, angle of attack and angle of sideslip would be helpful and is reproduced here, from flight dynamics-I, for ready reference.

#### I) Body axes system

To formulate and solve a problem in dynamics we need a system of axes. To define such a system, we note that an airplane is nearly symmetric in geometry and mass distribution about a plane which is called the plane of symmetry (Fig.1.7). This plane is used for defining the body axes system. Figure 1.8 shows a system of axes ($OX_bY_bZ_b$) fixed on the airplane which moves with the airplane and hence called body axes system. The origin 'O' of the body axes system is the center of gravity (c.g.) of the body which, by assumption of symmetry, lies in the plane of symmetry. The axis $OX_b$ is taken as positive in the forward direction. The axis $OZ_b$ is perpendicular to $OX_b$ in the plane of symmetry, positive downwards. The axis $OY_b$ is perpendicular to the plane of symmetry such that $OX_bY_bZ_b$ is a right handed system.

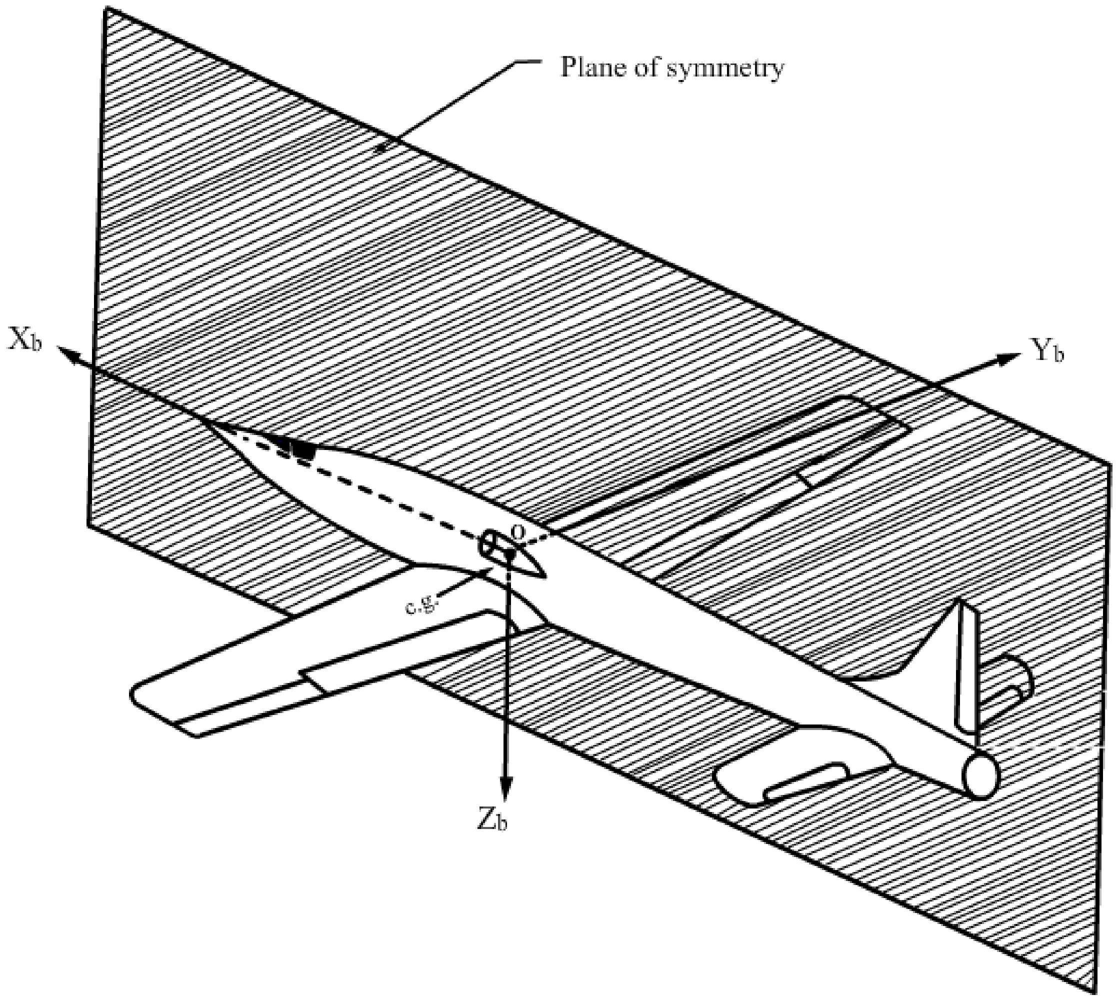



Fig.1.7 Plane of symmetry and body axis system

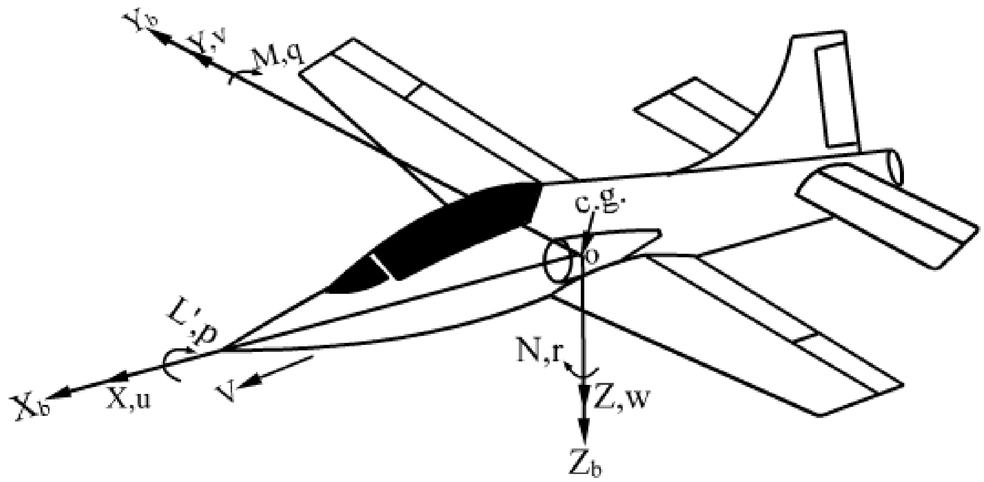



Fig.1.8 Body axes system, forces, moments and linear and angular velocities

Figure 1.8 also shows the forces and moments acting on the airplane and the components of linear and angular velocities. The quantity **V** is the velocity vector. The quantities X, Y, Z are the components of the resultant aerodynamic force, along $OX_b$, $OY_b$ and $OZ_b$ axes respectively. $L'$ , M, N are the rolling moment, pitching moment and yawing moment respectively about $OX_b$, $OY_b$ and $OZ_b$; the rolling moment is denoted by $L'$ to distinguish it from lift (L) . Figure 1.8 also shows the positive directions of $L'$, M and N. The convention is that an aerodynamic moment is taken positive in clock-wise sense when looking along the axis about which the moment is taken. u,v,w are the components , along $OX_b$, $OY_b$ and $OZ_b$ of the velocity vector (**V**). The angular velocity components are indicated by p,q,r.

## II) Earth fixed axes system

In flight dynamics a frame of reference attached to the earth is taken as a Newtonian frame (Fig.1.9).

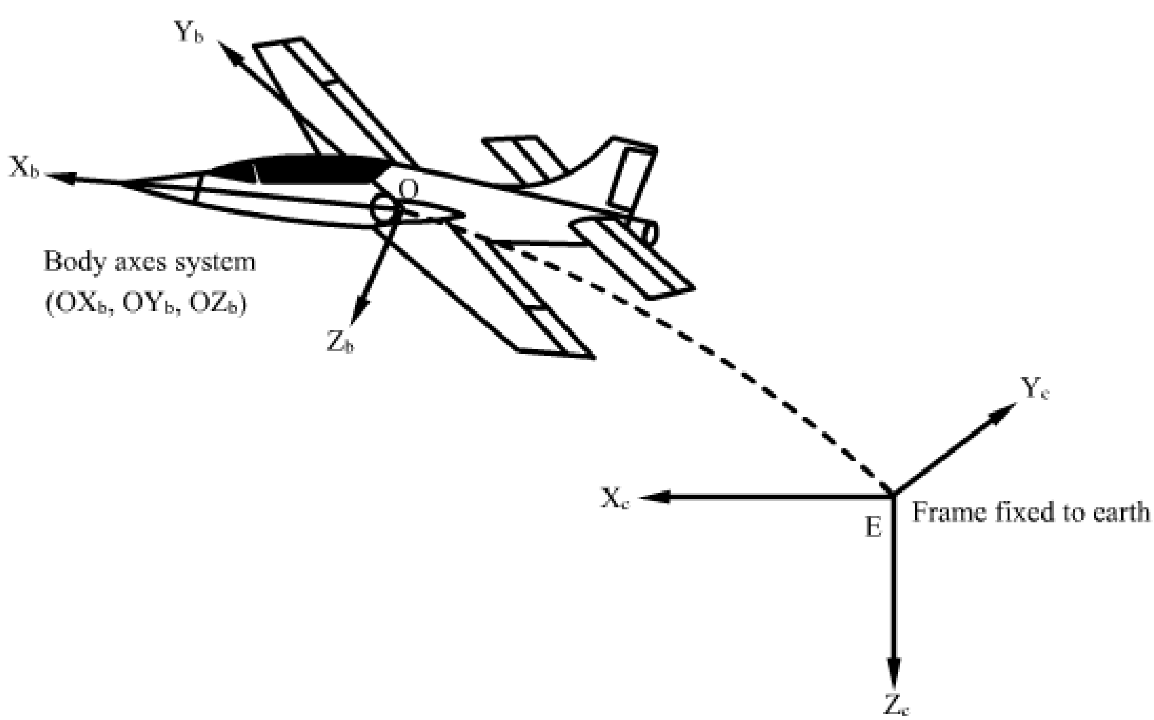



Fig.1.9 Earth fixed and body fixed co-ordinate systems

## III) Attitude

In this course the airplane is treated as a rigid body. In section 1.5.2 of flight dynamics-I, it has been shown that a rigid body has a six degrees of freedom and hence, six coordinates are needed to describe the position of the airplane with respect to an earth fixed system. In flight dynamics, the six corrdinates employed to prescribe the position are (a) the three coordinates describing the instantaneous position of the c.g. of the airplane with respect to the earth fixed system and (b) the attitude of the airplane described by the angular orientations of $OX_bY_bZ_b$ system with respect to the $OX_eY_eZ_e$ system. This is done with the help of Euler angles. In section 7.5.2 it is shown that to arrive at the $OX_bY_bZ_b$ system, we need to rotate the $EX_eY_eZ_e$ system through only three angles which are called Euler angles.

At this stage, simpler cases are considered. When an airplane climbs along a straight line its attitude is given by the angle '$\gamma$ ' between the axis $OX_b$ and the horizontal (Fig.1.10 ). When an airplane executes a turn, the projection of the $OX_b$ axis, in the horizontal plane makes an angle $\Psi$ with reference to fixed horizontal axis (Fig.1.11). When an airplane is banked, the axis $OY_b$ makes an angle $\varphi$ with respect to the horizontal and the axis $OZ_b$ makes an angle $\varphi$ with vertical (Fig.1.12).

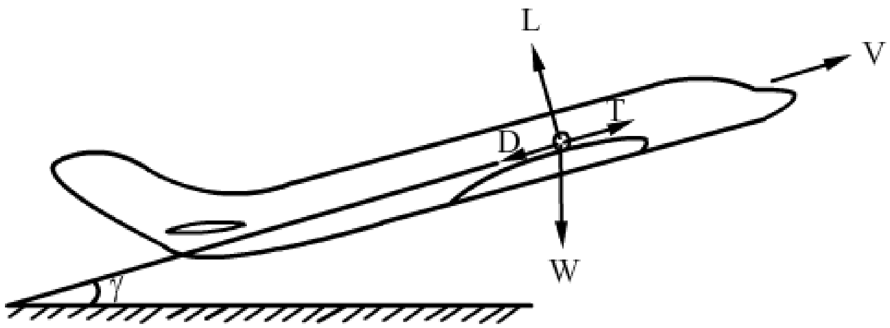
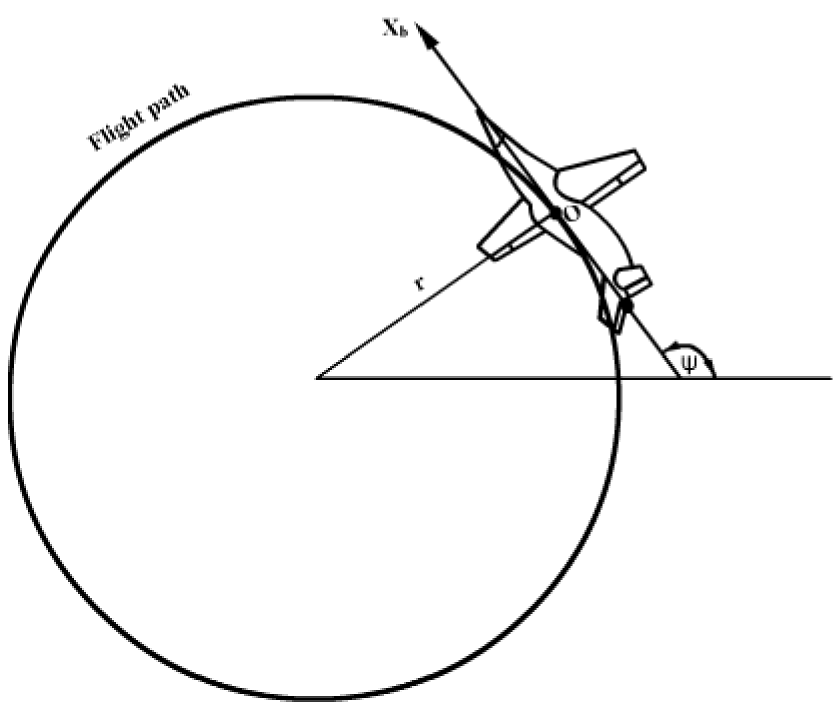



Fig.1.10 Airplane in a climb




Note : The flight path is circular in shape. Please adjust the resolution of your monitor so that the flight path looks circular.

Fig.1.11 Airplane in a turn – view from top

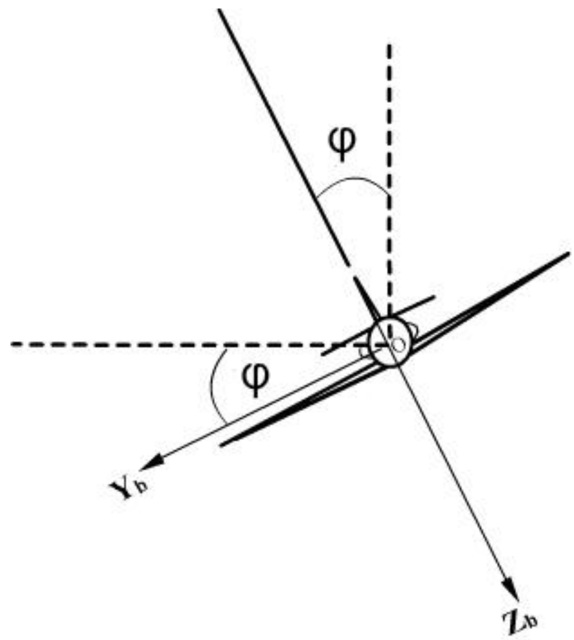



Fig.1.12 Angle of bank (φ)

## IV) Flight path:

The flight path, also called the trajectory, means the path or the line along which the c.g. of the airplane moves. The tangent to this curve at a point gives the direction of flight velocity at that point on the flight path. The relative wind is in a direction opposite to that of the flight velocity.

## V) Angle of attack and angle side slip

The concept of the angle of attack of an airfoil is well known. While discussing the forces acting on an airfoil, we take the chord of the airfoil as the reference line and the angle between the chord line and the relative wind is the angle of attack (α). The aerodynamic forces namely lift (L) and drag (D), produced by the airfoil, depend on the angle of attack (α) and are respectively perpendicular and parallel to relative wind direction (Fig.1.13).

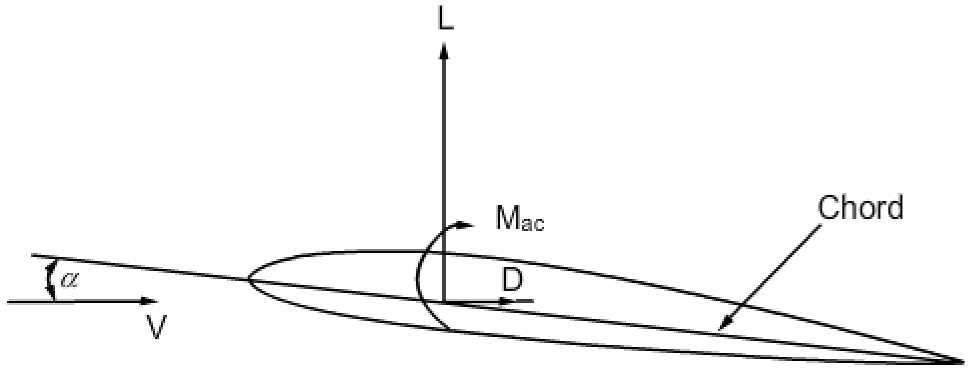



Fig.1.13 Angle of attack and forces on a airfoil

In the case of an airplane, the flight path, as mentioned earlier, is the line along which c.g. of the airplane moves. The tangent to the flight path is the direction of flight velocity (**V**). The relative wind is in a direction opposite to the flight velocity. If the flight path is confined to the plane of symmetry, then the angle of attack would be the angle between the relative wind direction and the fuselage reference line (FRL) or $OX_b$ axis (see Fig.1.14). However, in a general case the velocity vector (**V**) will have components both along and perpendicular to the plane of symmetry. The component perpendicular to the plane of symmetry is denoted by 'v'. The projection of the velocity vector in the plane of

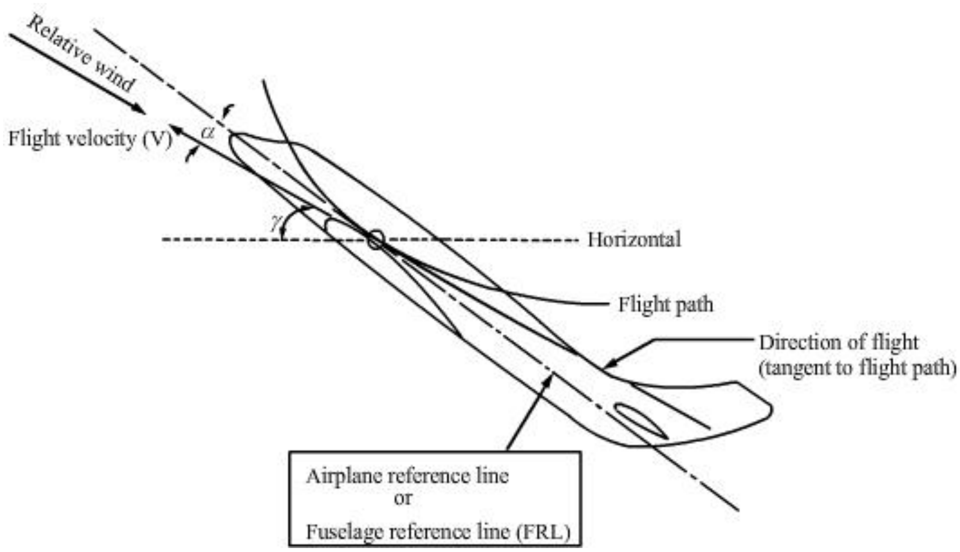



Fig.1.14 Flight path in the plane of symmetry

symmetry would have components u and w along $OX_b$ and $OZ_b$ axes (Fig.1.15). With this background, the angle of sideslip and angle of attack are defined below.

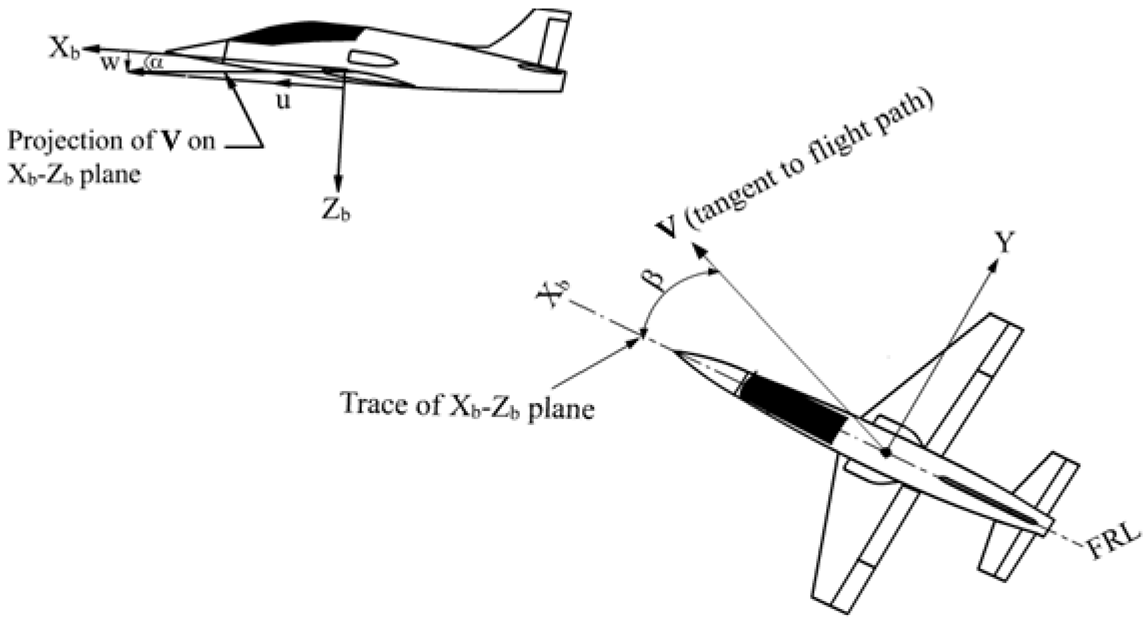



Fig.1.15 Velocity components in a general case and definition of angle of attack and sideslip

The angle of sideslip (β) is the angle between the velocity vector (**V**) and the plane of symmetry i.e.

$\beta = \sin^{-1} (v/ |\mathbf{V}|)$; where **|V|** is the magnitude of **V**.

The angle of attack (α) is the angle between the projection of velocity vector (**V**) in the $X_B$ - $Z_B$ plane and the $OX_b$ axis or

$$\alpha = \tan^{-1} \frac{w}{u} = \sin^{-1} \frac{w}{\sqrt{|\mathbf{V}|^2 - v^2}} = \sin^{-1} \frac{w}{\sqrt{u^2 + w^2}}$$

**Remark:**

It is easy to show that, if V denotes magnitude of the velocity (**V**) , then

$u = V \cos\alpha \cos\beta$ , $v = V \sin\beta$; $w = V \sin\alpha \cos\beta$ .