Chapter- 9

Performance analysis – V- Manoeuvres

(Lectures 28 to 31)

Keywords : Flights along curved path in vertical plane – loop and pull out ; load factor ; steady level co-ordinated-turn - minimum radius of turn, maximum rate of turn; flight limitations ; operating envelop; V-n diagram

Topics

Chapter 9

Lecture 28

Performance analysis V – Manoeuvres – 1

Topics

9.1 Introduction

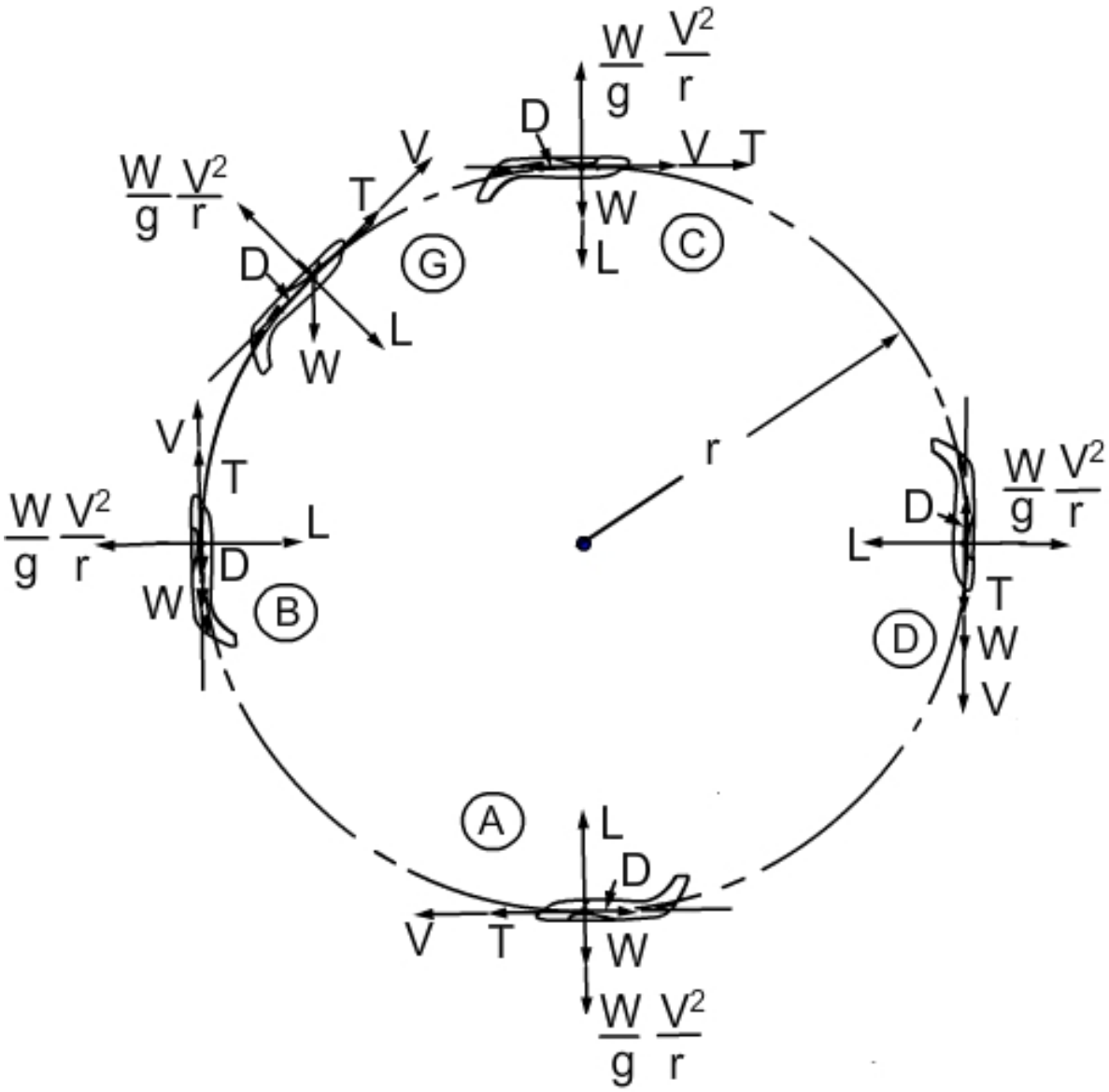
Flight along a curved path is known as a manoeuvre. In this flight the radial acceleration is always present even if the tangential acceleration is zero. For example, from particle dynamics (Ref.1.2) we know that when a body moves with constant speed along a circle it is subjected to a radial acceleration equal to (V^2 / r) or $\omega^2 r$ where, V is the speed, r is the radius of curvature of the path and ω is the angular velocity ($\omega = V / r$). In a general case, when a particle moves along a curve it has an acceleration along the tangent to the path whose magnitude is equal to the rate of change of speed (**V**) and an acceleration along the radius of curvature whose magnitude is (V^2 / r). Reference 1.1, chapter 1 may be referred to for details. In order that the body has these accelerations a net force, having components along these directions, must act on the body. For example, in the simpler case of a body moving with constant speed along a

circle, there must be a centripetal force of magnitude $m\omega^2 r$ in the radially inward direction; m is the mass of the body.

For the sake of simplicity, the motions of an airplane along curved paths confined to either the vertical plane or the horizontal plane, are only considered here. The flight along a closed curve in a vertical plane is refered to as loop and that in the horizontal plane as turn. Reference 2.1 and Ref. 1.12, chapter 2, may be referred to for various types of loops and turns. However, the simpler cases considered here illustrate important features of these flights.

9.2 Flight along a circular path in vertical plane (simplified loop)

Consider the motion of an airplane along a circular path of radius r with constant speed V. The forces acting on the airplane at various points of the flight path are shown in Fig.9.1. Note also the orientation of the airplane at various points and the directions in which D and L act; in a flat earth model W always acts in the vertically downward direction.



Note : The flight path is circular. Please adjust the resolution of your monitor so that the flight path looks circular.

Fig.9.1 Flight along a loop with constant radius and speed

(Note: The quantity $\frac{W}{g}\frac{V^2}{r}$ is the magnitude of the inertia force at various points)

9.2.1 Equations of motion in a simplified loop

The equations of motion, when the airplane is at specified locations, can be written down as follows.

At point A : $T - D = 0 \; ; \; L - W = \frac{WV^2}{g\,r}$ (9.1)

At Point B: $T - D - W = 0 \; ; \; L = \frac{W}{g}\frac{V^2}{r}$ (9.2)

At point C : $T - D = 0 \; ; \; L + W = \frac{W}{g}\frac{V^2}{r}$ (9.3)

At point D : $T - D + W = 0 \; ; \; L = \frac{WV^2}{gr}$ (9.4)

At a general point G the equations of motion are:

$$T - D - W\sin\gamma = 0 \; ; \; L + W\cos\gamma = \frac{WV^2}{gr} \quad (9.5)$$

Note that the Eqs. (9.1) to (9.4) for points A, B, C and D can be obtained from Eqs. (9.5) by substituting γ as 180^o, 90^o, 0^o and 270^o respectively.

Remarks:

i) If the tangential velocity is not constant during the loop then the first equation of Eqs.(9.5) would become:

$T - D - W \sin\gamma = (W / g)\, a$, where $a = dV / dt$ (9.6)

ii) From Eqs. (9.1 to 9.5) it is observed that the lift required and the thrust required during a loop with constant ‘r’ and ‘V’ change rapidly with time. It is difficult for the pilot to maintain these values and the actual flight path is somewhat like the one shown in Fig. 9.2.

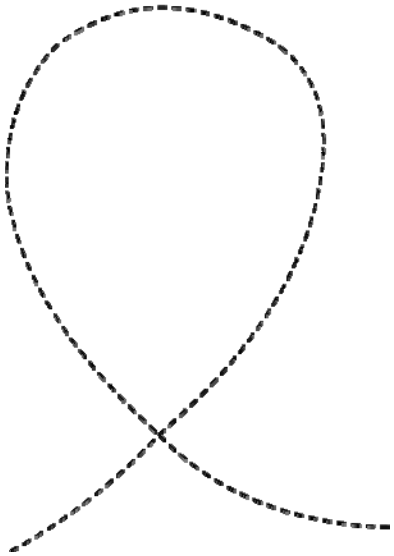


Fig.9.2 Shape of a normal loop

9.2.2 Implications of lift required during simplified loop

It is observed, that at the bottom of the loop i.e. point 'A' in Fig. 9.1, the lift required is equal to $W + \frac{WV^2}{gr}$ or $L = W\left(1+\frac{V^2}{gr}\right)$. The term (V^2 / gr) could be much larger than 1 and the lift required in a manoeuvre could be several times the weight of the airplane. As an illustration, let the flight velocity be 100 m/s and the radius of curvature be 200 m, then the term (V^2 / gr) is equal to 5.1. Thus the total lift required at point 'A' is 6.1 W. In order that an airplane carries out the manoeuvres without getting disintegrated, its structure must be designed to sustain the lift produced during manoeuvres. Secondly, when lift produced is high, the drag would also be high and the engine must produce adequate output. Further, lift coefficient cannot exceed C_{Lmax}, and as such no manoeuvre is possible at $V= V_{stall}$.

9.2.3 Load factor

The ratio of the lift to the weight is called 'Load factor' and is denoted by 'n' i.e.

$$n = (L / W) \qquad (9.7)$$

A flight with a load factor of n is called 'ng' flight. For example, a turn (see example 9.2) with load factor of 4 is referred to as a 4g turn. In level flight, n equals 1 and it is a 1g flight.

Higher the value of n, greater would be the strength required of the structure and consequently higher structural weight of the airplane. Hence, a limit is prescribed for the load factor to which an airplane can be subjected to. For example, the civil airplanes are designed to withstand a load factor of 3 to 4 and the military airplanes to a load factor of 6 or more. The limitation on the military airplane comes from the human factors namely, a pilot subjected to more than 6g may black out during the manoeuvre which is an undesirable situation.

To monitor the load factor, an instrument called 'g-meter' is installed in the cockpit.

9.2.4 Pull out

The recovery of an airplane from a dive or a glide is called a pull out (Fig. 9.3). The dive is an accelerated descent while the pull out phase can be regarded as a flight along an arc of a circle (See example 9.1).

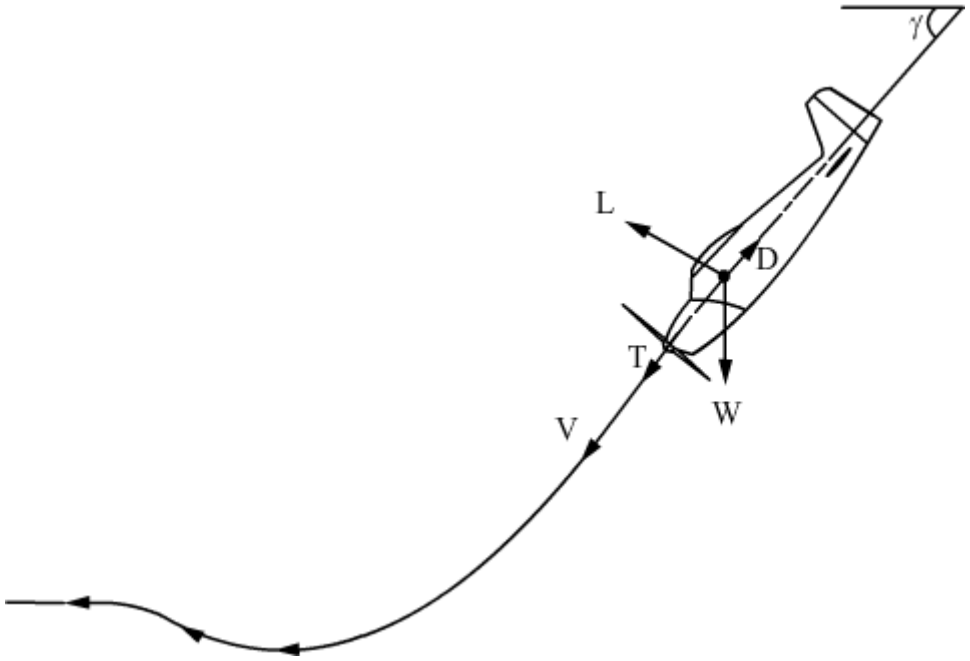


Fig.9.3 Pull out from dive

Example 9.1

An airplane with a wing area of 20 m^2 and a weight of 19,620 N dives with engine switched off, along a straight line inclined at 60^o to the horizontal. What is the acceleration of the airplane when the flight speed is 250 kmph? If the airplane has to pull out of this dive at a radius of 200 m, what will be the lift coefficient required and the load factor? Drag polar is given by: $C_D = 0.035 + 0.076 C_L^2$ and the manouevre takes place around an altitude of 2 km.

Solution:

From Fig. 9.3 the equations of motion in the dive can be written as follows.

$$L - W\cos\gamma = 0;\ W\sin\gamma - D = \frac{W}{g}a$$

$\gamma = 60^o$, Hence, $\cos\gamma = 0.5$ and $\sin\gamma = 0.866$

Consequently, L = 19620 x 0.5 = 9810 N

The drag of the airplane(D) can be obtained by knowing C_D which depends on C_L.

$$C_L = \frac{2L}{\rho S V^2}$$

V = 250 kmph = 69.4 m/s, ρ at 2 km = 1.0065 kg / m^3

Hence,

$$C_L = \frac{2 \times 9810}{1.0065 \times 20 \times 69.4^2} = 0.2024$$

Consequently, $C_D = 0.035 + 0.076 \times 0.2024^2 = 0.03811$

The drag $D = L\frac{C_D}{C_L} = 9810 \times \frac{0.03811}{0.2024} = 1847.3\text{N}$

Hence, $(W/g)\,a = W\sin\gamma - D = 19620 \times 0.866 - 1847.3 = 15144.1$ N

Or $a = 15144.1 \times 9.81 / 19620 = 7.57\ m/s^2$.

To obtain the lift required during the pull out, let us treat the bottom part of the flight path during the pull out as an arc of a circle.

From Eqs. (9.1) to (9.5), the lift required is maximum at the bottom of the loop and is given by:

$$L = W + \frac{WV^2}{gr} \text{ or } L = 19620 \times \left(1 + \frac{1}{9.81} \times \frac{69.4^2}{200}\right)$$

Or $L = 19620 \times 3.45$ Then,

$$C_L = \frac{19620 \times 3.45 \times 2}{1.0065 \times 20 \times 69.4^2} = 1.396$$

Remarks:

i) The maximum load factor in the above pull out is 3.45. The value of lift coefficient required is 1.396. This value may be very close to C_{Lmax} and the parabolic drag polar may not be valid.

ii) Since C_L cannot exceed C_{Lmax}, a large amount of lift cannot be produced at low speeds. Thus maximum attainable load factor ($n_{maxattainable}$) at a speed is:

$$n_{maxattainable} = (1/2)\, \rho\, V^2\, S\, C_{Lmax} / W$$

At stalling speed the value of n is only one.

9.3 Turning flight

When an airplane moves along an arc of a circle about a vertical axis then the flight is called a turning flight. When the altitude of the airplane remains constant in such a flight, it is called a level turn. In order that a turning flight is possible, a force must act in the direction of the radius of curvature. This can be done by banking the airplane so that the lift vector has a component in the horizontal direction. It may be added that the side force produced by deflecting the rudder is not large. It also causes considerable amount of drag, which is undesirable.

9.3.1 Steady, level, co-ordinated-turn

If there is no tangential acceleration i.e. the flight speed is constant, then the flight is called a steady turn. If the altitude remains constant then the flight is

called a level turn. When the airplane executes a turn without sideslip, it is called co-ordinated-turn. In this flight the X-axis of the airplane always coincides with the velocity vector. The following two aspects may also be noted regarding the steady, level, co-ordinated-turn.

(a) The centripetal force needed to execute the turn is provided by banking the wing. The horizontal component of the lift vector provides the centripetal force and the vertical component balances the weight of the airplane. Hence, the lift in a turn is greater than the weight.

(b) An airplane executing a turn, does produce a sideslip.

Because of the aforesaid two factors, a pilot has to apply appropriate deflections of elevator and rudder to execute a co-ordinated-turn.

A co-ordinated-turn is also called 'Correctly banked turn'. In this chapter, the discussion is confined to the steady level, co-ordinated-turn.

9.3.2 Equations of motion in steady level co-ordinated-turn

The forces acting on an airplane in steady, level, co-ordinated-turn are shown in Fig.9.4. The equations of motion in such a flight can be obtained by resolving the forces in three mutually perpendicular directions.

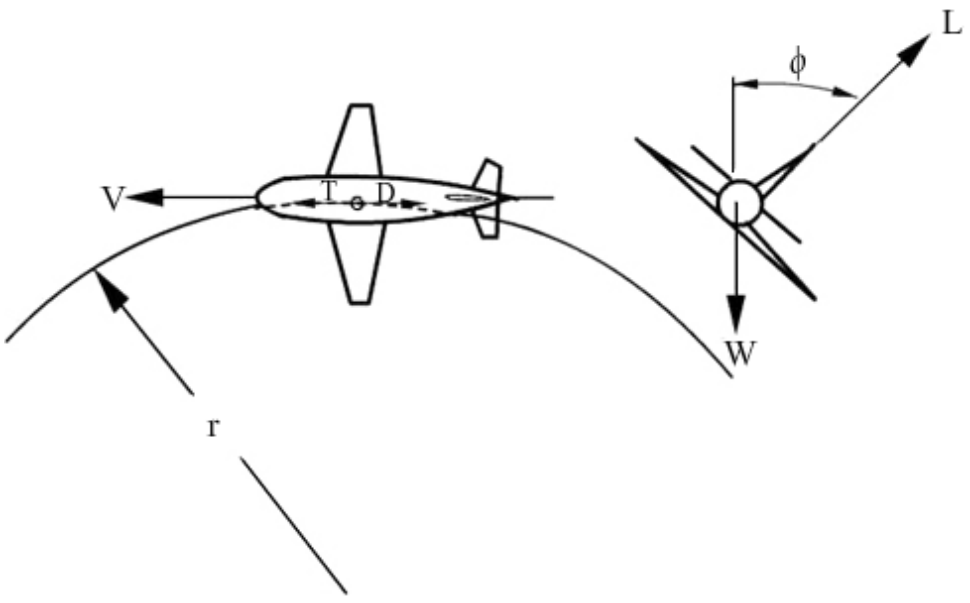


Fig.9.4 Turning flight

As the turn is a steady flight: T – D = 0 . (9.8)

As the turn is a level flight: W – L cos ϕ = 0. (9.9)

As the turn is co-ordinated which implied that, there is no unbalanced sideforce.

$$L \sin\phi = \frac{W}{g}\frac{V^2}{r} \tag{9.10}$$

where ϕ is the angle of bank and r is the radius of turn.

Remarks:

i) From the above equations it is noted that L = W / cos ϕ. Hence, in a turn L is larger than W. Consequently, drag will also be larger than that in a level flight at the same speed. The load factor n is equal to 1/ cos ϕ and is higher than 1.

ii) From Eqs. (9.9) and (9.10), the radius of turn r is given by:

$$r = \frac{W}{g}\frac{V^2}{L\sin\phi} = \frac{V^2}{g\tan\phi} \tag{9.11}$$

Noting that, $\cos\phi = \frac{1}{n}$ gives $\tan\phi = \sqrt{n^2-1}$ and

$$r = \frac{V^2}{g\sqrt{n^2-1}} \tag{9.11a}$$

The rate of turn, denoted by ($\dot{\psi}$), is given by:

$$\dot{\psi} = \frac{V}{r} = V / \frac{V^2}{g\tan\phi} = \frac{g\tan\phi}{V} \tag{9.12}$$

Noting $\tan\phi = \sqrt{n^2-1}$ gives :

$$\dot{\psi} = \frac{g\sqrt{n^2-1}}{V} \tag{9.12a}$$

(iii) In some books, the radius of turn is denoted by ‘R’. However, herein the letter ‘R’ is used to denote range, and to avoid confusion, the radius of turn is denoted by ‘r’.

Example 9.2

An airplane has a jet engine which produces a thrust of 24,525 N at sea level. The weight of the airplane is 58,860 N. The wing has an area of 28 m^2, zero-lift angle of – 2.2^o and a slope of lift curve of 4.6 per radian. Find (a) the radius of a correctly banked 4g level turn at the altitude where σ = 0.8 and the wing incidence is 8^o, (b) time required to turn through 180^o and (c) thrust required in the manoeuvre if the drag coefficient at this angle of attack be 0.055.

Solution:

The given data are: W = 58860 N, S = 28 m^2, α = 8^o,α_{0L} = -2.2^o,

$\frac{dC_L}{d\alpha}$ = 4.6 per radian = $\frac{4.6}{180}$ x 2π per degree = 0.083 per degree,

allowable n = 4 and T = 24525 N at sea level.

Consequently,

$$C_L = \frac{dC_L}{d\alpha}(\alpha - \alpha_{0L}) = 0.0803\,(8 + 2.2) = 0.82$$

In a 4g turn $L = 4W = 1/2\ \rho V^2 S C_L$

$$\text{Hence, } V = (2L/\rho S C_L)^{1/2} = \left(\frac{2 \times 4 \times 58860}{1.225 \times 0.8 \times 28 \times 0.82}\right)^{1/2} = 144.6 \text{ m/s.}$$

$$\cos\phi = \frac{1}{n} = \frac{1}{4} \text{ or } \phi = 75^0 31'$$

Hence, $\tan\phi = 3.873$

$$\text{Consequently, } r = \frac{V^2}{g\tan\phi} = \frac{(144.6)^2}{9.81 \times 3.873} = 550.3\,m$$

$$\text{Rate of turn} = \dot{\psi} = \frac{V}{r} = \frac{144.6}{550.3} = 0.2627 \text{ rad/s}$$

Hence, time to turn through 180^o is equal to $\frac{\pi}{0.2627} = 11.95\,s$

The thrust required = $T_r = 1/2\ \rho V^2 S C_D$

$= (1/2) \times 1.225 \times 0.8 \times 144.6^2 \times 28 \times 0.055 = 15786$ N

Answers : (a) Radius of correctly banked turn = 550.3 m, (b) time required to turn through 180^0 = 11.95 s and (c) thrust required during turn = 15,786 N

Remark:

The thrust available is given as 24525 N at sea level. If the thrust available is assumed to be roughly proportional to ($\sigma^{0.7}$), the thrust available at the chosen altitude would be $24525 \times 0.8^{0.7}$ = 20978 N. This thrust is more than the thrust required during the turn and the flight is possible.