Chapter 9

Dynamic stability analysis – III – Lateral motion - 2

Lecture 34

Topics

9.6 Lateral-directional response of general aviation airplane (Navion)

9.7 Approximations to modes of lateral motion

9.8 Two parameter stability diagram for lateral motion

9.6. Lateral-directional response of general aviation airplane (Navion)

Reference 1.12 chapter 6 presents the response of general aviation airplane discussed in example 9.1. Substituting the values of stability derivatives the square matrix in Eq.(9.19), for this airplane, is:

$$\begin{bmatrix} -0.2543 & 0 & -53.64 & -9.80665 & 0 \\ -0.298 & -8.402 & 2.193 & 0 & 0 \\ 0.0838 & -0.3498 & -0.7608 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

The five roots in this case are: $\lambda = 0, -0.0089, -8.4345$ and $-0.4867 \pm i\,2.3314$. Following may be noted.

A) The time to half amplitude ($t_{1/2}$) of spiral mode is:

$t_{1/2} = 0.693 / 0.0089 = 77.9$ s.

B) The period of the Dutch roll is:

$2\pi / 2.3314 = 2.69$ s

C) The time to half amplitude for the Dutch roll is: 0.693 / 0.4867 =1.42 s

D) Number of cycles for half amplitude of Dutch roll is:

1.42 / 2.69 = 0.53 cycles

The response to various disturbances are briefly discussed below.

A) Disturbance in sideslip:

Figure 9.3a shows the response to a disturbance of $\Delta\beta = 5^0$ at time t = 0. It is seen that the Dutch roll mode persists for the first few cycles and , as a consequence, various quantities show oscillatory variations. Subsequently, the variables tend to zero except $\Delta\psi$ which assumes a small value (Fig.9.3a). This is due to the zero root.

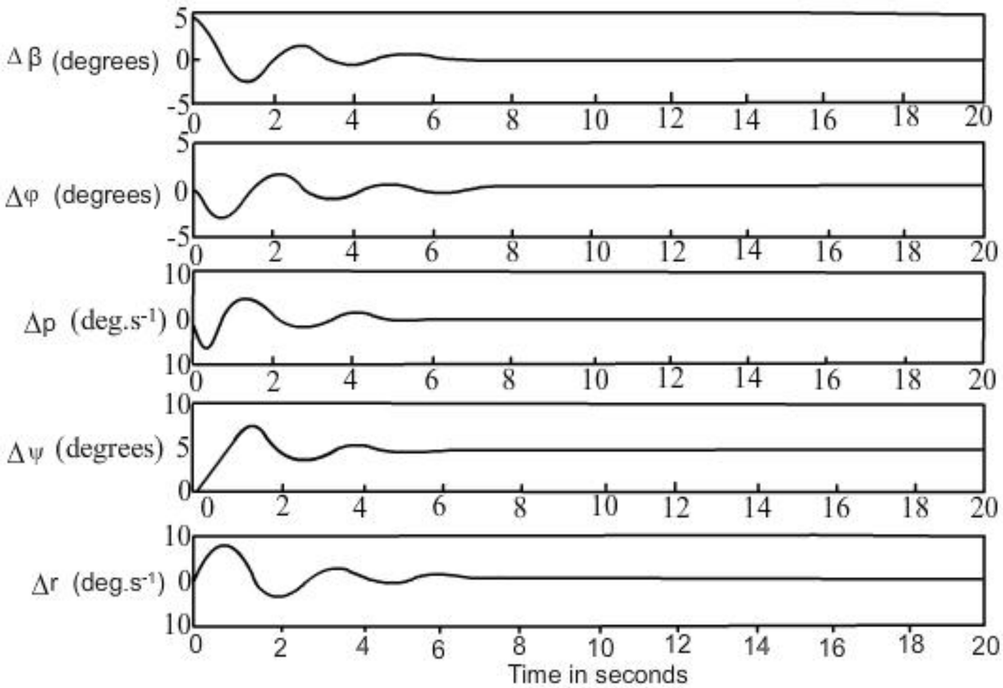

Fig. 9.3a Lateral-directional response of the general aviation airplane-disturbance in Δβ (Adapted from Ref.1.12, chapter 6 with permission from American Institute of Aeronautics and Astronautics, Inc.)

B) Figure 9.3b shows the response to a disturbance of bank angle $\Delta\varphi = 5^0$ at t = 0. The Dutch roll mode causes initial oscillations which decay rapidly. However, the spiral mode has a low damping and it takes long time to decay. It is again noticed that the yaw angle ($\Delta\psi$) attains a steady state value due to one of the roots being zero.

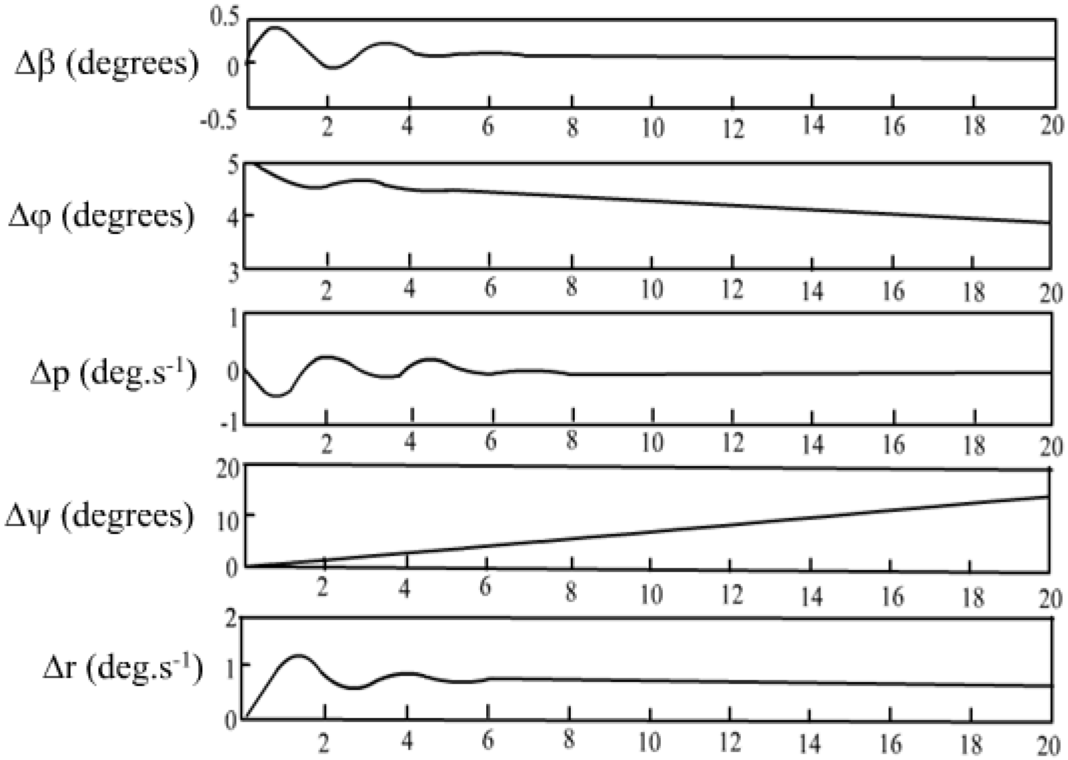

Fig. 9.3b Lateral-directional response of the general aviation airplane-disturbance in Δφ (Adapted from Ref.1.12, chapter 6 with permission from American Institute of Aeronautics and Astronautics, Inc.)

C) Figure 9.3c shows the response to a disturbance in rate of roll, $\Delta p = 0.2$ rad s^{-1} (11.5 deg s^{-1}) at t = 0. It is seen that the roll rate goes to zero without oscillation.

However sideslip, bank angle and the rate of yaw display oscillatory motions. The angle of yaw gradually attains a non-zero value again due to the presence of zero root.

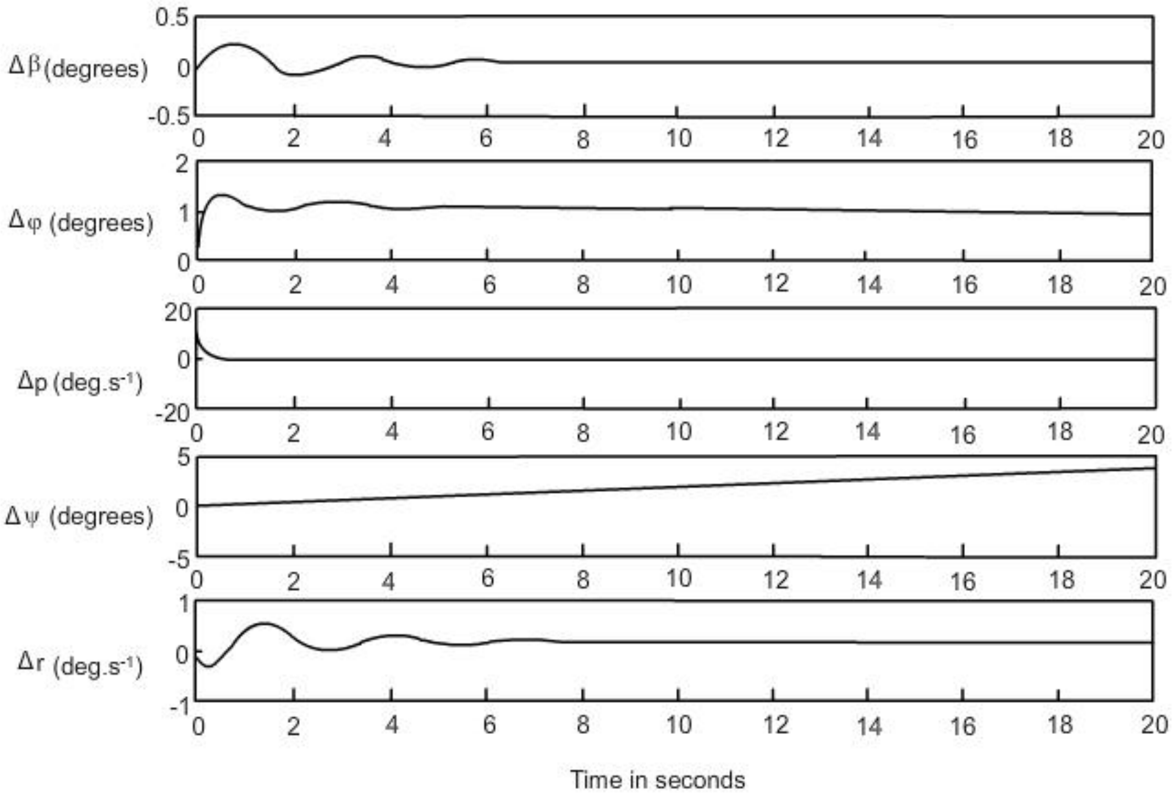

Fig. 9.3c Lateral-directional response of the general aviation airplane-disturbance in Δp (Adapted from Ref.1.12, chapter 6 with permission from American Institute of Aeronautics and Astronautics, Inc.)

D) Figure 9.3d shows response to a disturbance in rate of yaw of $\Delta r = 0.2$ rad s^{-1} (11.5 deg s^{-1}) at t = 0. It is observed that the response is similar to that in the other cases.

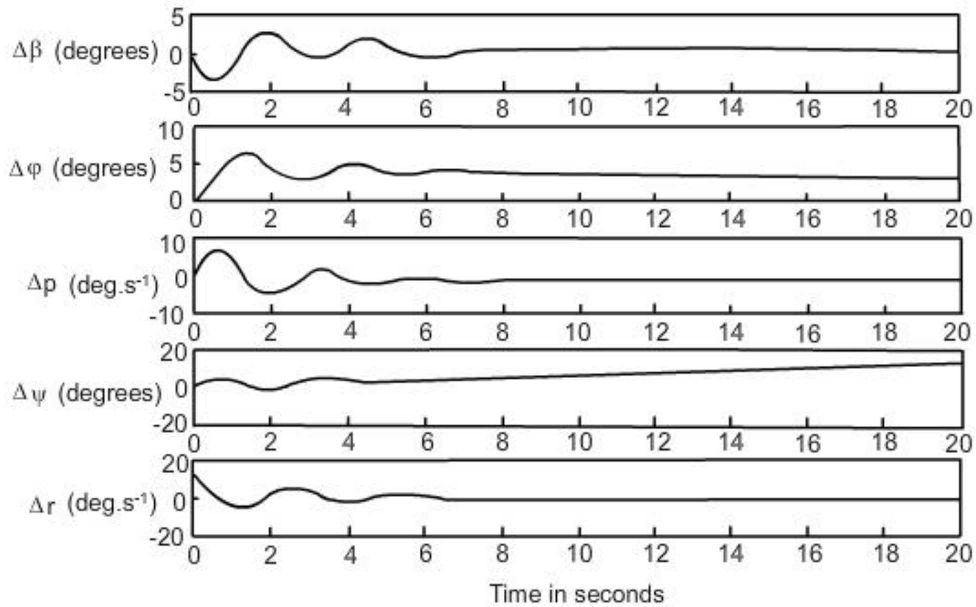

Fig.9.3d Lateral-directional response of the general aviation airplane- disturbance in Δr (Adapted from Ref.1.12, chapter 6 with permission from American Institute of Aeronautics and Astronautics, Inc.)

Remarks:

i) Since, the governing equations are linear, the response to a combination of small initial disturbances can be obtained by summing up the responses given above.

ii) For a video clip on motion of Basset airplane during Dutch roll, see: http://in.youtube.com/watch?v=vqVQ_s8XL6s

9.7 Approximations to modes of lateral motion

Reference 1.1, chapter 5 and Ref.1.12, chapter 6 present approximate analyses of spiral and Dutch roll modes. The important conclusions of the approximate analyses are as follows.

a) The spiral root is approximately equal to:

$$\lambda_{spiral} \approx \frac{L'_\beta N_r - L'_r N_\beta}{L'_\beta} \tag{9.23}$$

Noting that L'_β is proportional to dihedral effect and is negative, for λ_{spiral} to be negative, the numerator in Eq.(9.23) should be positive or ($L'_\beta N_r$ - $L'_r N_\beta$)> 0 or (L'_β / N_β) > (L'_r /N_r).

b) The imaginary part of Dutch roll mode is approximately given as :

$$\omega_{Dutch} \approx (N_\beta)^{1/2} \tag{9.24}$$

Remark:

The results of approximate analyses of lateral motion are not as good as those in the case of approximation to longitudinal motion. Hence, it is recommended that exact roots be obtained in the lateral case.

9.8 Two parameter stability diagram for lateral motion

The area of the vertical tail (S_v) and the dihedral angle (Γ) are the two parameters which can be controlled by the designer to achieve desired level of lateral stability. The parameter S_v mainly affects N_β and Γ affects L'_β . Hence, these parameters are suited for a two parameter stability diagram. Based on Ref.1.5, chapter 7, the following two parameters are chosen.

$$n_v = N_v / (½ \rho u_0 Sb) \text{ and } l_v = L'_v / (½ \rho u_0 Sb) \tag{9.25}$$

Note : The quantity l_v in Eq.(9.25) need not be confused with the distance between c.g. and aerodynamic centre of vertical tail (Fig.5.5) which is also denoted by l_v .

Following observations are made.

I) Fig.9.4a shows a two parameter stability diagram for the lateral motion when C_L = 0.1 which may indicate the higher end of the flight velocity range. It may be noted that:

a) The parameter μ_2 in the figure is defined as:

$$\mu_2 = 2 m / (\rho Sb) \tag{9.26}$$

b) The spiral boundary denotes the case when E_1 in the characteristic equation for lateral motion (Eq.9.13) is zero. This is like the divergence boundary in longitudinal case (Fig.8.8).

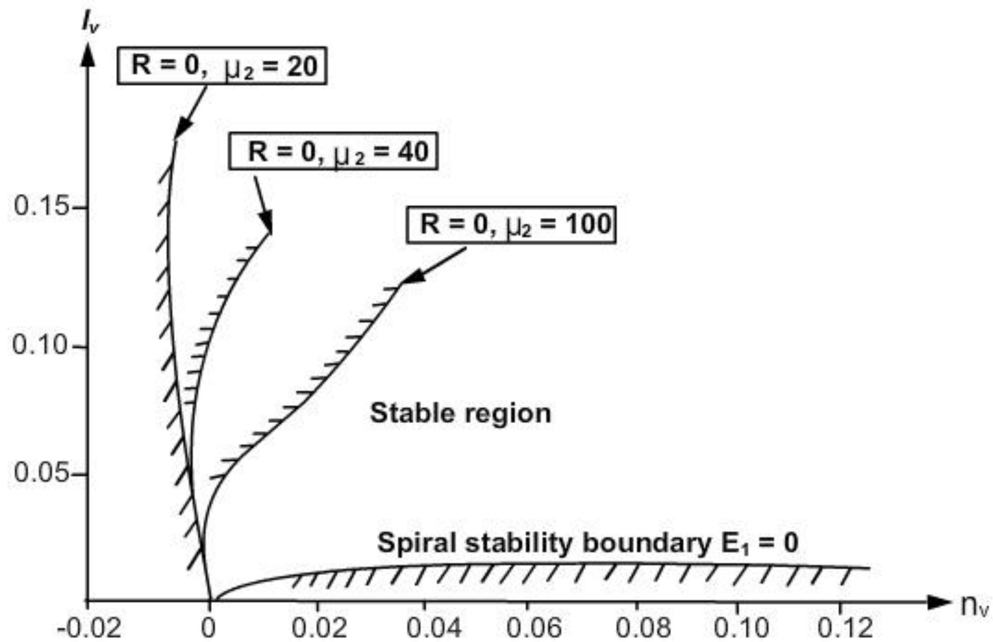

Fig.9.4a Two parameter stability diagram for lateral motion- C_L=0.1 (Adapted from Ref.1.5, Pt.II chapter 7 and in turn from Ref.4.4, with permission from controller of HM stationary office)

c) The Routhian boundary is obtained when the Routhian (R) is zero. In the lateral case this boundary depends on μ_2. From Eq.(9.25) it is seen that μ_2 increases as ρ decreases (or the altitude increases).

From Fig.9.4a it is observed that the Routhian boundary shifts closer to the spiral boundary as μ_2 increases.

II) Figure 9.4b shows the two parameter stability diagram for lateral motion when C_L = 1.0, which may indicate the low speed end of the flight velocity range. It is interesting to note that for μ_2 = 40 and 100, major part of the spiral boundary lies above the Routhian boundary. This indicates that either the spiral mode or the Dutch roll mode would be unstable.

The above results indicate the need for investigating the stability of the airplane under various combinations of flight velocity, altitude and weight. In situations where one of the roots is unstable, the earlier practice was to choose S_v and Γ such that Dutch roll mode is stable. The present method is to use automatic control to alter the level of stability. Section 10.3.1 be referred for a brief discussion on automatic control.

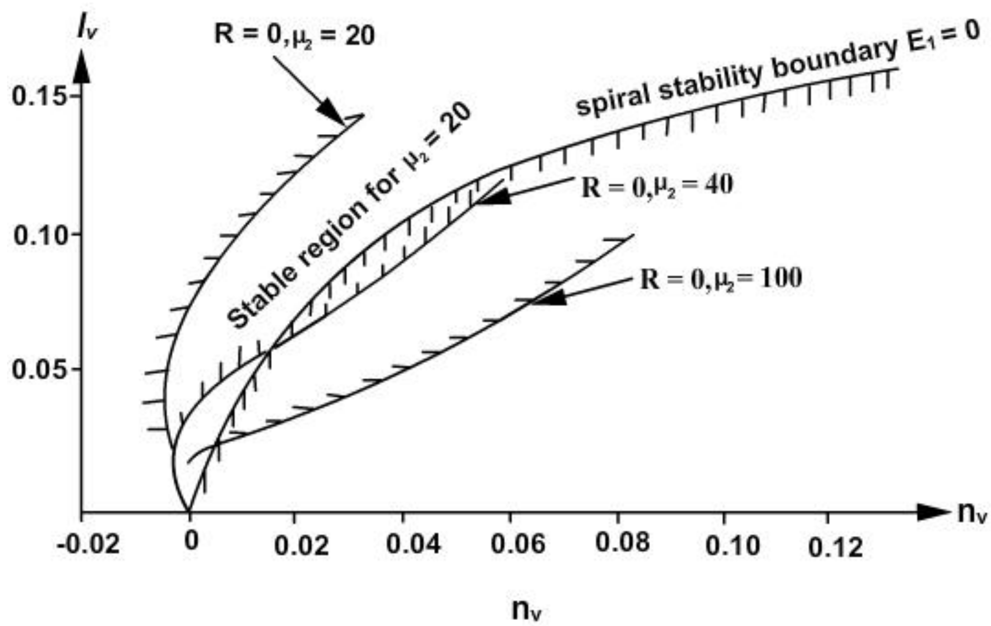

Fig. 9.4b Two parameter stability diagram for lateral motion- C_L = 1.0 (Adapted from Ref.1.5, Pt.II chapter 7 and in turn from Ref.4.4, with permission from controller of HM stationary office)

General Remark:

See Appendix ‘C’ for evaluation of the stability derivatives and the solution of stability quartic for Boeing 747 airplane.