Chapter 2

Lecture 8

Longitudinal stick–fixed static stability and control – 5

Topics

2.6 Contributions of power plant to C_{mcg} and $C_{m\alpha}$

The contributions of power plant to C_{mcg} and $C_{m\alpha}$ have two aspects namely direct contribution and indirect contribution.

2.6.1 Direct contribution of power plant to C_{mcg} and $C_{m\alpha}$

The direct contribution appears when the direction of the thrust vector does not coincide with the line passing through the c.g.(Fig.2.24). The direct contribution is written as :

$$M_{cgp} = T \times Z_p \tag{2.59}$$

where, T is the thrust and Zp is the perpendicular distance of thrust line from FRL; positive when c.g. is above thrust line.

In non-dimensional form Eq.(2.59) is expressed as:

$C_{mcgp} = M_{cgp}/(½\ \rho V^2 S\bar{c})$ (2.60)

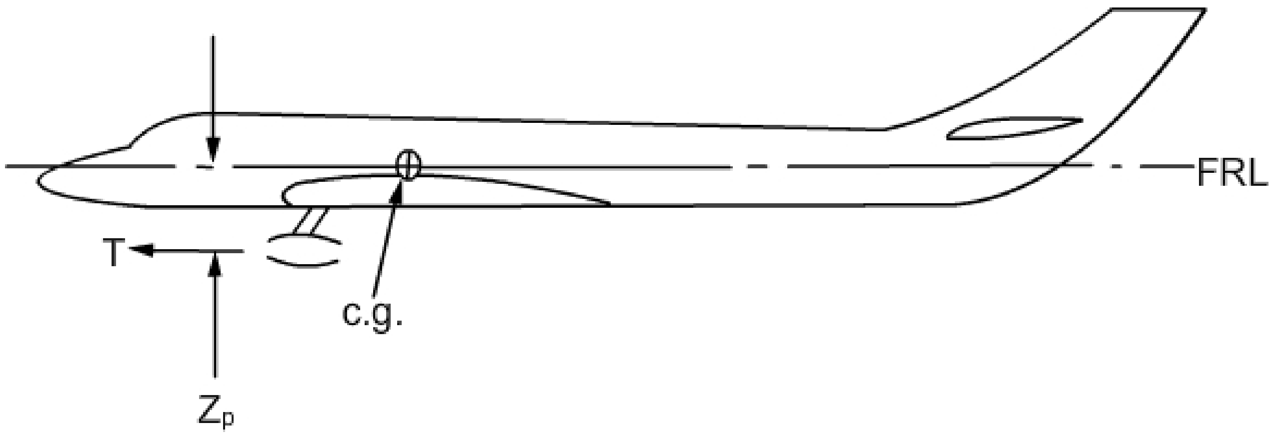

Fig.2.24 Contribution of thrust to C_{mcg}

The thrust required varies with flight speed and altitude. Hence, C_{mcgp} would vary with flight condition. However, the thrust setting does not change during the disturbance and hence, there is no contribution to $C_{m\alpha}$. This fact is also mentioned in Ref.1.9. p.506.

The contribution to $C_{m\alpha}$ comes from another cause. Consider a propeller at an angle of attack as shown in Fig.2.25. The free stream velocity (V) is at an angle (α)to the propeller axis. As the air stream passes through the propeller it leaves in a nearly axial direction. This change of direction results in a normal force (N_p) in addition to the thrust (T).

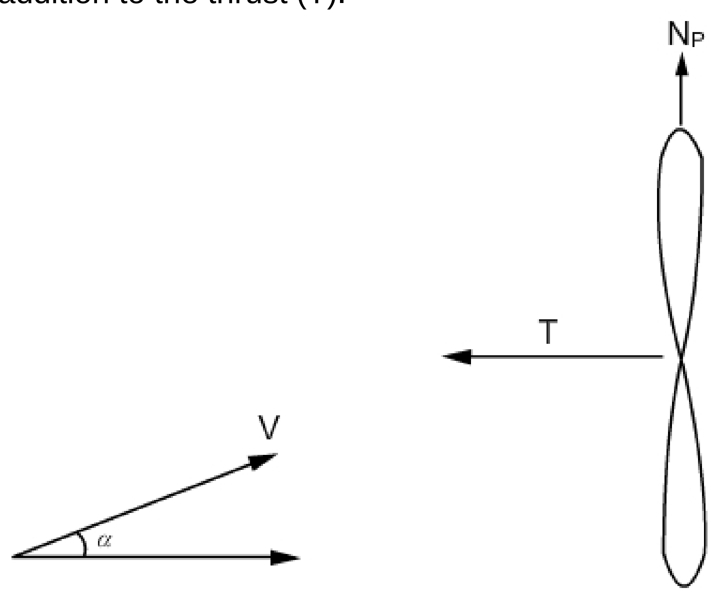

Fig.2.25 Propeller at angle of attack

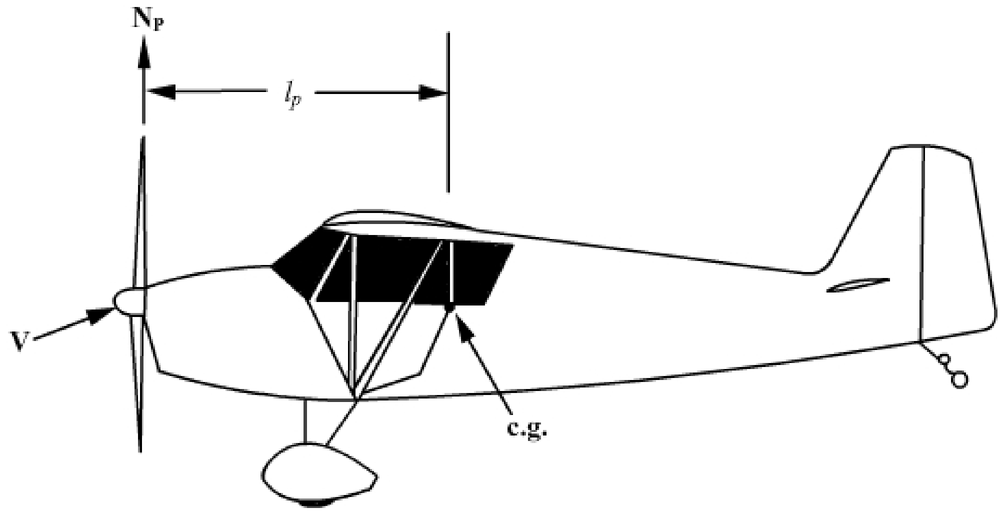

Fig.2.26 Contribution to $C_{m\alpha}$ from normal force due to propeller

N_p acts at distance l_p from the c.g. (Fig.2.26) and hence, produces a moment N_p x l_p. The value of N_p depends on the angle of attack of the propeller and hence the term N_p x l_p depends on α. This will contribute to $C_{m\alpha}$. $C_{m\alpha}$ due to normal force depends on many factors like thrust setting, number of blades in the propeller and advance ratio.

Remarks:

i) It is evident from Fig.2.26 that when the propeller is ahead of c.g., the contribution to $C_{m\alpha}$ due to normal force would be positive or destabilizing. In a pusher airplane, where the propeller is near the rear end of the airplane, the contribution of normal force to $C_{m\alpha}$ will be negative and hence stabilizing.

ii) In the case of a jet engine at an angle of attack, the air stream enters the intake at that angle and its direction has to change as the stream passes through the engine. This change of direction will also produce a normal force N_p and consequently contribute to $C_{m\alpha}$.

2.6.2 Indirect contributions of power plant to C_{mcg} and $C_{m\alpha}$

The effect of propeller on the horizontal tail has been discussed in section 2.4.3. In the case of an airplane with a jet engine, the exhaust expands in size as it moves downwards and entrains the surrounding air. This would induce an angle to the flow; the induced angle would be positive in the region below the jet and negative in the region above the jet. In military airplanes where the engine is located in rear fuselage the engine exhaust would affect the horizontal tail, generally located above the rear fuselage, by inducing a downwash in addition to that due to wing. This effect will also come into picture in case of passenger airplanes with rear mounted engines. To alleviate this, the horizontal tail is mounted above the vertical tail (see configurations of Boeing MD-87 and Gulf stream V in Ref.2.3).

Remarks:

i) The contribution of engine depends also on the engine power setting which in turn depends on flight condition or C_L. Hence, the level of stability ($C_{m\alpha}$) will depend on C_L and also will be different when engine is off or on.

ii) It is difficult to accurately estimate the effects of power on $C_{m\alpha}$. A rough estimate would be (Ref.1.7, chapter 5) :

$$(dC_m / dC_L)_p = 0.04 \text{ or } C_{m\alpha p} = 0.04C_{L\alpha} \tag{2.60a}$$

2.7 General Remarks:

2.7.1 Slope of lift curve ($C_{L\alpha}$) and angle of zero lift (α_{0L}) of the airplane:

Let, L denote lift of airplane. Then, $L = L_{wb}+L_t$.

For airplanes with large aspect ratio wings (A>5), the lift of the wing body combination is approximately equal to lift produced by the gross wing i.e.. $L_{wb} \approx L_w$

Noting that $L_t = ½\rho V_t^2 S_t C_{L\alpha t} (\alpha - \varepsilon + i_t)$ and $L_w = ½ \rho V^2 S C_{Lw}$; the slope of the lift curve of the airplane ($C_{L\alpha}$) can be written as :

$$C_{L\alpha} = C_{L\alpha w} + \eta (S_t/S)C_{L\alpha t} \{1-(d\varepsilon/d\alpha)\} \tag{2.60b}$$

Reference 1.8 b gives expressions for corrections to obtain $C_{L\alpha wb}$ from $C_{L\alpha w}$ (see also Appendix C section 5).

2.7.2 Angle of zero lift (α_{0L}) for airplane:

Assuming that the wing is set such that during cruise the angle of attack of the airplane (α_{cr}) is zero, the lift coefficient during cruise (C_{Lcr}) can be written as :

$$C_{Lcr} = C_{L\alpha} (\alpha_{cr} - \alpha_{0L}) = C_{L\alpha} (0 - \alpha_{0L})$$

$$\text{Hence, } \alpha_{0L} = - C_{Lcr} / C_{L\alpha} \tag{2.60c}$$

2.8 C_{mcg} and $C_{m\alpha}$ of entire airplane

The important result of the last few sections can be recapitulated as follows.

$$C_{mcg} = (C_{mcg})_w + (C_{mcg})_f + (C_{mcg})_n + (C_{mcg})_p + (C_{mcg})_{ht} \tag{2.12}$$

$$C_{m\alpha} = (C_{m\alpha})_w + (C_{m\alpha})_f + (C_{m\alpha})_n + (C_{m\alpha})_p + (C_{m\alpha})_{ht} \tag{2.13}$$

The wing Contribution is:

$$C_{mcgw} = C_{macw} + C_{Lw}\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right) \tag{2.17}$$

$$\begin{aligned} C_{Lw} &= C_{L\alpha w}(\alpha + i_w - \alpha_{0Lw}) \\ &= C_{L0w} + C_{L\alpha w}\,\alpha \;;\; C_{L0w} = C_{L\alpha w}(i_w - \alpha_{0Lw}) \end{aligned} \tag{2.18}$$

$$C_{mcgw} = C_{macw} + C_{L0w}\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right) + C_{L\alpha w}\;\alpha\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right) \tag{2.19}$$

$$C_{m0w} = C_{macw} + C_{L0w}\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right) \tag{2.19a}$$

$$(C_{m\alpha})_w = C_{L\alpha w}\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right) \tag{2.20}$$

The tail Contribution is:

$$C_{mcgt} = -V_H\,\eta\,C_{Lt} \tag{2.37}$$

$$C_{Lt} = C_{L\alpha t}\,\alpha_t + C_{L\delta e}\,\delta_e + C_{L\delta t}\,\delta_t \tag{2.38}$$

$$\alpha_t = \alpha - \varepsilon + i_t = \alpha_w - i_w - \varepsilon + i_t \tag{2.39}$$

$$\varepsilon = \varepsilon_0 + \frac{d\varepsilon}{d\alpha}\alpha;\; \varepsilon_0 = \frac{d\varepsilon}{d\alpha}(i_w - \alpha_{0Lw}) \tag{2.41}$$

$$C_{Lt} = C_{L\alpha t}\left\{i_t - \varepsilon_0 + \alpha\left(1 - \frac{d\varepsilon}{d\alpha}\right)\right\} + C_{L\delta e}\,\delta_e + C_{L\delta t}\,\delta_t \tag{2.46}$$

$$C_{mcgt} = -V_H\,\eta\,C_{L\alpha t}\,\{i_t - \varepsilon_0 + \alpha\left(1 - \frac{d\varepsilon}{d\alpha}\right) + \tau\,\delta_e + \tau_{tab}\delta_t\} \tag{2.47}$$

$$\tau = \frac{C_{L\delta e}}{C_{L\alpha t}}\;;\;\tau_{tab} = \frac{C_{L_{\delta_t}}}{C_{L_{\alpha_t}}}$$

$$(C_{m\alpha t})_{stick\text{-}fixed} = -V_H\,\eta\,C_{L\alpha t}\left(1 - \frac{d\varepsilon}{d\alpha}\right) \tag{2.50}$$

The contributions of fuselage, nacelle and power are expressed together as:

$$(C_m)_{f,n,p} = (C_{m0})_{f,n,p} + (C_{m\alpha})_{f,n,p}\ \alpha \tag{2.61}$$

Substituting various expressions in Eqs.(2.12) and (2.13) gives:

$$C_{mcg} = C_{m0} + C_{m\alpha}\,\alpha + C_{m\delta e}\,\delta_e \tag{2.62}$$

$$C_{m0} = C_{macw} + C_{L0W}\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right) + (C_{m0})_{f,n,p} - V_H\ \eta\ C_{L\alpha t}\{i_t - \varepsilon_0 + \tau_{tab}\,\delta_t\} \tag{2.63}$$

$$C_{m\delta e} = -V_H\,\eta\,C_{L\alpha t}\,\tau \tag{2.64}$$

$$(C_{m\alpha})_{stick\ fixed} = C_{L\alpha w}\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right) + (C_{m\alpha})_{f,n,p} - V_H\,\eta\,C_{L\alpha t}\left(1 - \frac{d\varepsilon}{d\alpha}\right) \tag{2.65}$$

Typical contributions of the individual components and their sum, namely C_{mcg} for a low subsonic airplane are shown in Fig.2.27. The details of the calculations are given in example 2.4.

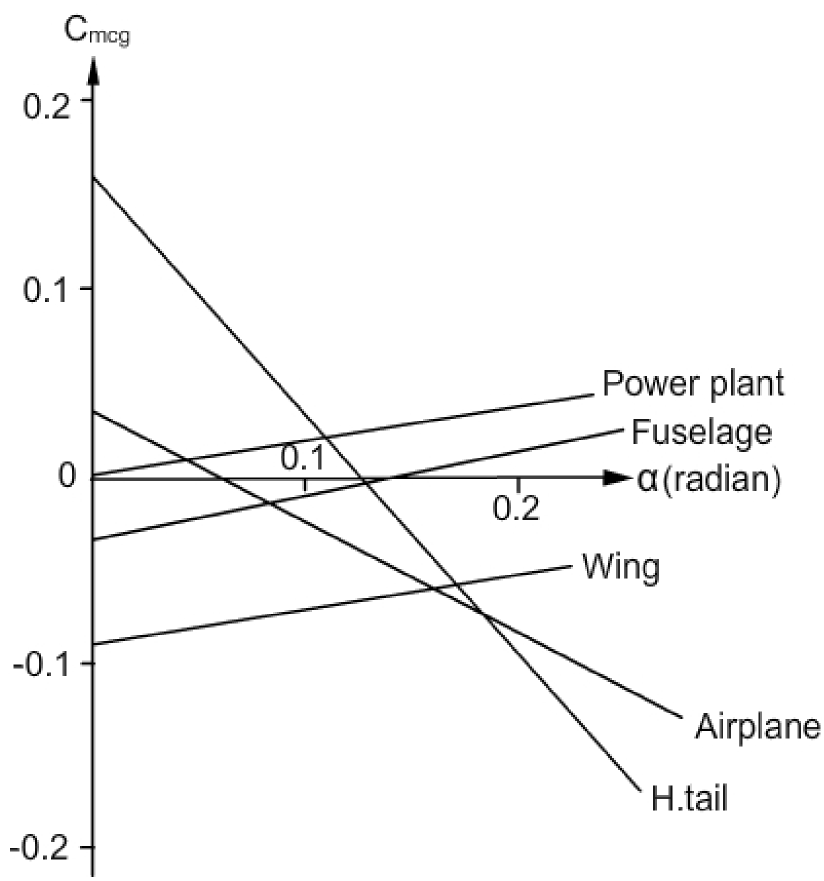

Fig.2.27 C_{mcg} vs α for a low subsonic airplane

Following observations can be made in this case.

(a)C_{mow} has an appreciable negative value.

(b)$C_{m\alpha w}$ depends on the product of $C_{L\alpha w}$ and $\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right)$. In the case considered in example 2.4, the c.g. is at $0.295\bar{c}$ and the a.c. is at $0.25\bar{c}$. Since, c.g. is aft of the aerodynamic centre, the contribution of wing is destabilizing (Fig.2.27).

(c) C_{mof} has small negative value and $C_{m\alpha f}$ has small positive value, indicating a slight destabilizing contribution from fuselage (Fig.2.27).

(d) C_{mot} is positive and $C_{m\alpha t}$ has a large negative value (Fig.2.27).

(e) The line corresponding to the sum of all the contribution (wing+ fuselage+ power+tail) is the C_{mcg} vs α curve for the whole airplane. The contribution of nacelle is ignored. It is seen that the large negative contribution of tail renders $C_{m\alpha}$ negative and the airplane is stable.

2.9 Stick-fixed neutral point

It may be pointed out that the c.g. of the airplane moves during flight due to consumption of fuel. Further, the contribution of wing to $C_{m\alpha}$ depends sensitively on the location of the c.g. as it is proportional to $\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right)$. When the c.g. moves aft, x_{cg} increases and the wing contribution becomes more and more positive. There is a c.g. location at which $(C_{m\alpha})_{\text{stick-fixed}}$ becomes zero. This location of c.g. is called the stick-fixed neutral point. In this case, the airplane is neutrally stable. Following Ref.1.1 this location of the c.g. is denoted as x_{NP}. If the c.g. moves further aft, the airplane will become unstable. The C_m vs. α curves for the statically stable, neutrally stable and unstable cases are schematically shown in Fig.2.28.

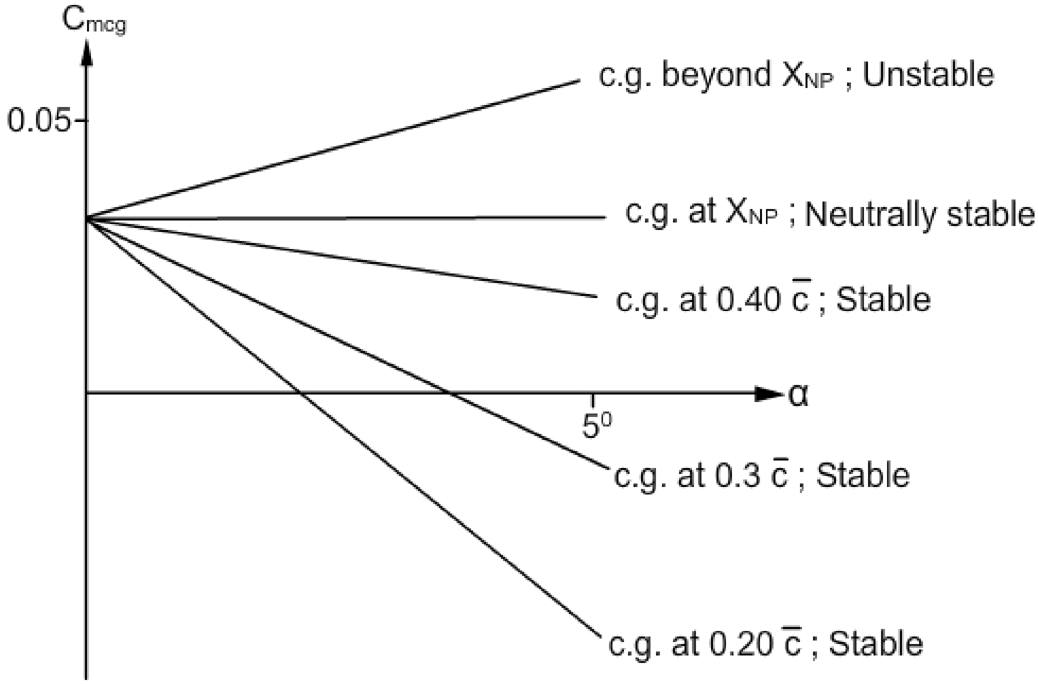

Fig.2.28 Changes in static stability with movement of c.g. (Schematic)

An expression for x_{NP} can be obtained by putting $C_{m\alpha} = 0$ and $x_{cg} = x_{NP}$, in Eq.(2.65) i.e.

$$0 = C_{L\alpha w}\left(\frac{x_{NP}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right) + (C_{m\alpha})_{f,n,p} - V_H \eta C_{L\alpha t}\left(1 - \frac{d\varepsilon}{d\alpha}\right) \tag{2.66}$$

$$\text{Hence, } \frac{x_{NP}}{\bar{c}} = \frac{x_{ac}}{\bar{c}} - \frac{1}{C_{L\alpha w}}\left\{(C_{m\alpha})_{f,n,p} - V_H \eta C_{L\alpha t}\left(1 - \frac{d\varepsilon}{d\alpha}\right)\right\} \tag{2.67}$$

Example 2.4 illustrates the steps involved in arriving at the neutral point.

2.9.1 Neutral point power-on and power-off

The contribution of power is generally destabilizing and hence, the airplane will be more stable when engine is off. In other words, x_{NP} power off is behind x_{NP} power on.

2.10 Static margin

Noting the definition of $\frac{x_{NP}}{\bar{c}}$ from Eq.(2.67), the Eq.(2.65) can be rewritten as :

$$(C_{m\alpha})_{stick\text{-}fixed} = C_{L\alpha w}\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{NP}}{\bar{c}}\right) \tag{2.68}$$

Thus, $(C_{m\alpha})_{\text{stick-fixed}}$ is proportional to $\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{NP}}{\bar{c}}\right)$ and a term called static margin is defined as:

$$\text{Static margin} = \left(\frac{x_{NP}}{\bar{c}} - \frac{x_{cg}}{\bar{c}}\right) \tag{2.69}$$

Consequently, $(C_{m\alpha})_{\text{stick-fixed}} = -C_{L\alpha w}$ x (static margin) (2.70)

and $\left(\frac{dC_m}{dC_L}\right)_{\text{stick-fixed}}$ = -(static margin)

$$= \frac{1}{C_{L\alpha}}(C_{m\alpha})_{\text{stick-fixed}} \tag{2.71}$$

It may be noted that static margin, by definition, is positive for a stable airplane.

2.11 Neutral point as the aerodynamic centre of entire airplane

To explain the above concept, the derivation of the expression for neutral point in the Ref.1.10. chapter 2 is briefly described. The wing contribution (C_{mcgw}) is expressed as:

$$C_{mcgw} = C_{macw} + \alpha_w\, C_{L\alpha w}\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right)$$

The contributions of fuselage and nacelle are accounted for by treating them as changes in the following quantities: (a) pitching moment coefficient is changed from C_{macw} to C_{macwb}, (b) the angle of attack is changed from α_w to α_{wb}, (c) the slope of the lift curve is changed from $C_{L\alpha w}$ to $C_{L\alpha wb}$ and (d) aerodynamic centre is change from x_{ac} to x_{acwb}. The suffix 'wb' indicates combined effects of wing body and nacelle. Consequently,

$$C_{mcgwb} = C_{macwb} + \alpha_{wb}\, C_{L\alpha wb}\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{acwb}}{\bar{c}}\right)$$

The contribution of power is expressed as C_{mcgp}.

The contribution of the horizontal tail is expressed as :

$C_{mcgt} = -\bar{V}_H\, C_{Lt}$; note $\eta = 1.0$ (assumed)

Where, $\bar{V}_H = \frac{S_t}{S}\,\frac{\bar{l}_t}{\bar{c}}$

$\bar{l}_t$ = distance between the aerodynamic centre of the wing-body-nacelle combination (x_{acwb}) and the aerodynamic centre of the horizontal tail.

It is assumed that the C_L and $C_{L\alpha}$ of the airplane are approximately equal to C_{Lwb} and $C_{L\alpha wb}$ respectively. The expression for C_{mcg} can now be written as:

$$C_{mcg} = C_{macwb} + C_L \left(\frac{x_{cg}}{\bar{c}} - \frac{x_{acwb}}{\bar{c}}\right) - \bar{V}_H C_{Lt} + C_{mcgp} \tag{2.62a}$$

$$\text{or } C_{m\alpha} = C_{L\alpha} \left(\frac{x_{cg}}{\bar{c}} - \frac{x_{acwb}}{\bar{c}}\right) - \bar{V}_H C_{L\alpha t} + C_{m\alpha p} \tag{2.65a}$$

The neutral point, x_{NP} , is given by:

$$\frac{x_{NP}}{\bar{c}} = \frac{x_{acwb}}{\bar{c}} - \frac{1}{C_{L\alpha}} (C_{m\alpha p} - \bar{V}_H C_{L\alpha t}) \tag{2.67a}$$

$$\text{or } C_{m\alpha} = C_{L\alpha} \left(\frac{x_{cg}}{\bar{c}} - \frac{x_{NP}}{\bar{c}}\right) = - C_{L\alpha} \times (\text{static margin}) \tag{2.70a}$$

It may be recalled that the aerodynamic centre of an aerofoil is the point about which the pitching moment is constant with angle of attack. Similarly, the aerodynamic centre of the wing (x_{ac}), by definition, is the point about which C_{macw} is constant with angle of attack. With this background, the quantity x_{acwb} can be called as the aerodynamic centre of the wing - body - nacelle combination. Further, when the c.g. is at neutral point, $C_{m\alpha}$ is zero or C_{mg} is constant with α. This may be the reason Ref.1.10, chapter 2 refers the neutral point as the aerodynamic centre of the entire airplane.

Remark:

There are some differences in the expressions on the right hand sides of Eq.(2.67) and (2.67a) and Eq.(2.70) and (2.70a). These differences are due to slight difference in treatment of the contributions of individual components. The differences in Eq.(2.70) and (2.70a) can be reconciled by noting that for airplanes with large aspect ratio wings , $C_{L\alpha} \approx C_{L\alpha w}$. Reference 1.12, chapter 3 also mentions of this approximation to $C_{L\alpha}$. It may be recalled that expression for slope of lift curve of the airplane is obtained in subsection 2.7.1.

Reference 1.8b also expresses

$$C_{m\alpha} = \left(\frac{dC_m}{dC_L}\right) C_{L\alpha}$$

where, $C_{L\alpha}$ is the slope of the lift curve of the airplane and

$$\frac{dC_m}{dC_L} = \frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}$$

where, x_{ac} is the location of neutral point. Thereby treating neutral point as the aerodynamic centre of the airplane (see Appendix C section 5.3).